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Unbiased Estimation for the Contextual Effect of Duration of Adolescent Height Growth on Adulthood Obesity and Health Outcomes via Hierarchical Linear and Nonlinear Models

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Unbiased Estimation for the Contextual Effect of Duration of Adolescent Height Growth on Adulthood Obesity and Health Outcomes via Hierarchical Linear and Nonlinear Models

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at Virginia Commonwealth University

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May, 2012

Acknowledgements

The process in which we fondly have termed “dissertating” has been a long-term commitment requiring diligence, perseverance, and a large amount of support and contribution from those who surround me. Dr. Shumei Sun has financially supported my training at VCU with her grant work and provided me an unmatched setting to study and develop into a researcher. Dr. Yongyun Shin has been through every step of this dissertation alongside of me; and has directed the studies and provided insight, motivation and critique to keep my momentum in the forward direction throughout the process. My other committee members of Drs. Christine Schubert, Levent Dumenci, and Stephen Daniels have been instrumental in forming the contents and ideas displayed in this dissertation.

The faculty of the Biostatistics Department at VCU guided my coursework and learning so that I can be successful in this field. Yvonne Hargrove personally is responsible for me successfully meeting each requirement for this degree. Thank you all for your support.

Personally I was entertained and distracted in ways completely necessary for completing my doctorate by my fellow students and friends in Richmond, Va. My teammates in cycling always offered welcome excursions to refresh my mind throughout the toils of graduate studies. Caroline Carr has given the continuous support that I could not have completed this journey without.

Finally, my family has inspired me to continually strive to achieve great milestones throughout my entire life. Ellen (Mom), Tom (Holmes/H-Dog), Tommy (T$), Abbey, Brian and Liz Carrico, as well as Jeb Haley and Colin McLaren have been there unconditionally, and cannot be thanked enough for their efforts in pushing me to this success.

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Abbreviations

BMI	Body Mass Index
d.f.	Degrees of Freedom
EB	Empirical Bayes
FLS	Fels Longitudinal Study
GLLAMM	Generalized Linear Latent and Mixed Models
GLM	Generalized Linear Model
HGLM	Hierarchical Generalized Linear Model
HGLM2	Two-level Hierarchical Generalized Linear Model
HLM	Hierarchical Linear Model
HLM2	Two-level Hierarchical Linear Model
HLM3	Three-level Hierarchical Linear Model
HyperT	Hypertension
Int	Intercept
OGS	Onset of Growth Spurt
PHV	Peak Height Velocity
Var	Variance
Var Comp	Variance Component
WC	Waist Circumference
WCI	Waist Circumference Index

Abstract

UNBIASED ESTIMATION FOR THE CONTEXTAL EFFECT OF DURATION OF ADOLESCENT HEIGHT GROWTH ON ADULTHOOD OBESITY AND HEALTH OUTCOMES VIA HIERARCHICAL LINEAR AND NONLINEAR MODELS

By Robert John Carrico

A dissertation submitted in partial fulfillment to the requirements for the degree of Doctor of Philosophy at Virginia Commonwealth University

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Major Director: Yongyun Shin, Ph.D., Assistant Professor, Department of Biostatistics

This dissertation has multiple aims in studying hierarchical linear models in biomedical data analysis. In Chapter 1, the novel idea of studying the durations of adolescent growth spurts as a predictor of adulthood obesity is defined, established, and illustrated. The concept of contextual effects modeling is introduced in this first section as we study secular trend of adulthood obesity and how this trend is mitigated by the durations of individual adolescent growth spurts and the secular average length of adolescent growth spurts. It is found that individuals with longer periods of fast height growth in adolescence are more prone to having favorable BMI profiles in adulthood.

In Chapter 2 we study the estimation of contextual effects in a hierarchical generalized linear model (HGLM). We simulate data and study the effects using the higher level group sample mean as the estimate for the true mean versus using an Empirical Bayes (EB) approach (Shin and Raudenbush 2010). We study this comparison for logistic, probit, log-linear, ordinal

and nominal regression models. We find that in general the EB estimate lends a parameter estimate much closer to the true value, except for cases with very small variability in the upper level, where it is a more complicated situation and there is likely no need for contextual effects analysis.

In Chapter 3 the HGLM studies are made clearer with large-scale simulations. These large scale simulations are shown for logistic regression and probit regression models for binary outcome data. With repetition we are able to establish coverage percentages of the confidence intervals of the true contextual effect. Coverage percentages show the percentage of simulations that have confidence intervals containing the true parameter values. Results confirm observations from the preliminary simulations in the previous section of this paper, and an accompanying example of adulthood hypertension shows how these results can be used in an application.

Chapter 1

The Effect of the Duration of Adolescent Height Growth on Adulthood Obesity Outcomes

1.1. Introduction

In longitudinal studies of childhood growth, the information from individual trajectories in height growth is well documented. With serial height measurements taken at regular examination visits before, during, and after one's pubertal period investigators have studied either periods or pivotal points of height growth, and their relationship with health biomarkers during puberty or later in life. Large-scale surveys have recorded height data on large cohorts of children which have been used to obtain national standards in height growth features, such as onset of the pubertal growth spurt (OGS), peak height velocity (PHV), and ages at both OGS and PHV (Tanner and Whitehouse 1976, Abbassi 1998). It has been established that serial measurements of BMI, waist circumference, body fat, and fat distribution track from childhood into adulthood (Bao et al, 1995, 1996; Porkka et al, 1994; Guo et al, 2000; Serdula et al, 1993; Srinivasan et al, 1996; Whitaker et al, 1997).

Growth curves have also been used to establish national percentiles for children's growth in both height and weight and their respective first derivative velocity curves and individual's height acceleration curves (Ramsay, Altman, and Bock 1994). Different statistical methods have been used to analyze individual growth curves in height. These studies aimed to find stable estimates of PHV and onset of growth spurt (OGS), which could be used to infer physical attributes and biomarkers. For example, a triple logistic function has been used to model individual stature curves in which parameters represent features such as OGS and PHV. These estimates were then used to study how the timing of OGS and PHV affect cardiovascular and metabolic risk factors (Sun and Schubert 2009). The timing of OGS and PHV were shown to be risk factors for adult diseases such as type-2 diabetes and metabolic syndrome (Sun and Schubert 2009). Preece-Baines models using differential equations have been used to estimate these same points in growth (Guo and Siervogel 1992). Spline modeling and data smoothing techniques have been implemented in the same way (Ramsay, Altman, and Bock 1994). Polynomial models also have been fitted for height or BMI growth versus age (Philippaerts et. al. 2006, Guo et al, 2000). In these polynomial models, the inflexion points are interpreted as milestones in growth, such as OGS and PHV.

In this paper, we aim first to describe individual height growth as a time interval during which an individual grows above a certain threshold in growth rate or a time interval during which an individual grows between lower and upper thresholds in growth rate. We then study the impact of these thresholds and time intervals on biomarkers of health and obesity in adulthood, using the Fels Longitudinal Study (FLS, Roche, 1992). The FLS is unique in that it has measured individuals in 82 cohorts at regular scheduled examinations from birth through adulthood beginning in 1929 and is still collecting data on the offspring of the older cohort (Roche 1992). We are focusing on the time around the PHV, when an adolescent is growing steadily at a rapid rate. The FLS follows children into adulthood through regular adolescent visits in which height and other biomarkers are measured. Children are examined semi-annually to 18 years and adults are examined biennially for a lifetime. Therefore, the FLS data allow us to link children's growth characteristics such as growth rate in height --- the polynomial function in the growth rate in particular --- with obesity related biomarkers in adulthood. The obesity biomarkers of interest will be BMI and waist circumference. An extensive sample such as the FLS allows the studying of childhood characteristics and then linking them to adulthood health problems. Many of these adulthood health issues have been found to originate in childhood (Grundy 2002; Berenson and Srinivasan, 2001). Obesity is cited as the most common and costly nutritional problem in the United States (Kuczmarski et al, 1994; NHLBI Report, 2007).

Our approach is to extract from the FLS data the duration that an individual grows at or above a threshold rate in cm/year which defines a length of time that includes the PHV. This length of time includes the main feature of height growth rate (velocity of height growth) and allows us to study its impact on adulthood obesity biomarkers. We hypothesize that the duration of rapid growth influences risk factors for adult obesity. The first step is to document the velocity in height growth of each individual during childhood. Then the features of the height velocity trajectories will be studied in connection with their impact on adulthood health outcomes.

1.2. Methods

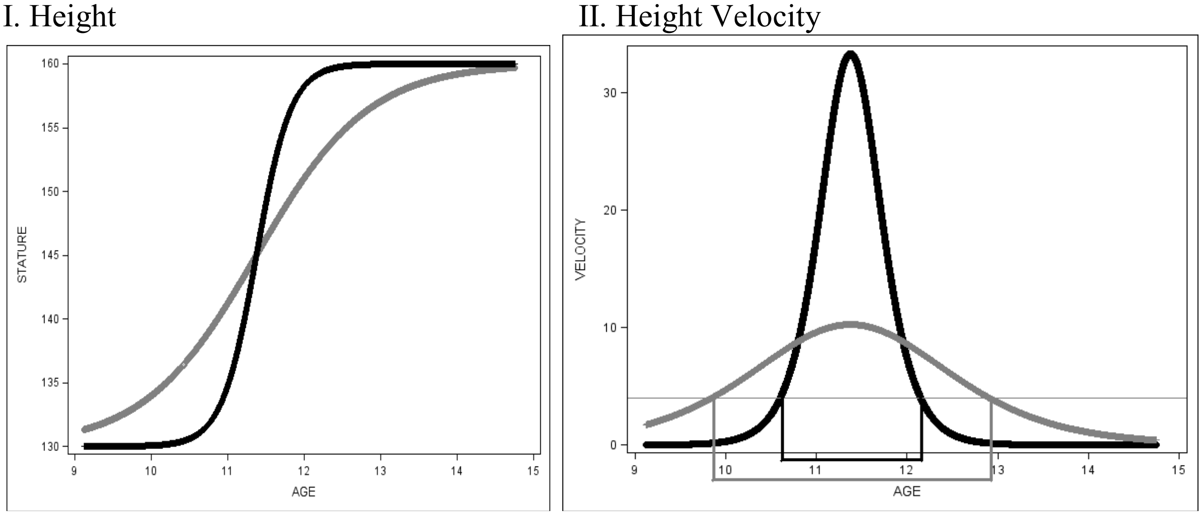
1.2.1 Description of Height Growth Curve Derived Variables and Sample

One of the most intensely studied features of an individual height growth is the peak of height velocity. To measure the velocity surrounding and including the peak, we first compute the derivatives of each individual's serial height data from the FLS, the annual growth rate in cm. We use all individuals who have height data from which derivatives can be calculated, as well as adulthood serial data. This selection resulted in 442 males and 407 females. These individuals' height data from age 8 to 18 years were used to compute the annual velocity in height growth in adolescence. To make growth rates comparable between individuals, we scaled the derivatives by the adult height for each individual. Thus the height growth is a percentage change relative to adulthood height, which eliminates the bias from the fact that taller people have higher peak height velocity (Schubert and Sun 2009).

From these measures, we compute the length of time a person spends with scaled height velocity greater than some threshold *d* % of adulthood height per year. The choice of the appropriate *d* that may explain individual differences in adult health biomarkers is the main goal of this analysis, and the calculation from this threshold will be referred to as Δ_d. Sun and Schubert 2009 reported that the timing of the PHV is predictive of adulthood obesity. Our method considers the entire length of the adolescent growth spurt, not just the time about PHV. Figure 1 compares the height distance and height velocity curves of two hypothetical individuals (one with black lines, the other with grey) with differing trajectories. The horizontal axis is age in years in both plots while the vertical axes represent height and the yearly growth rate in centimeters, respectively.

Figure 1. Mean height curves for two individuals with different height growth patterns but peak height velocity for both individuals approximately at 11.5 years old.

I. Height   II. Height Velocity

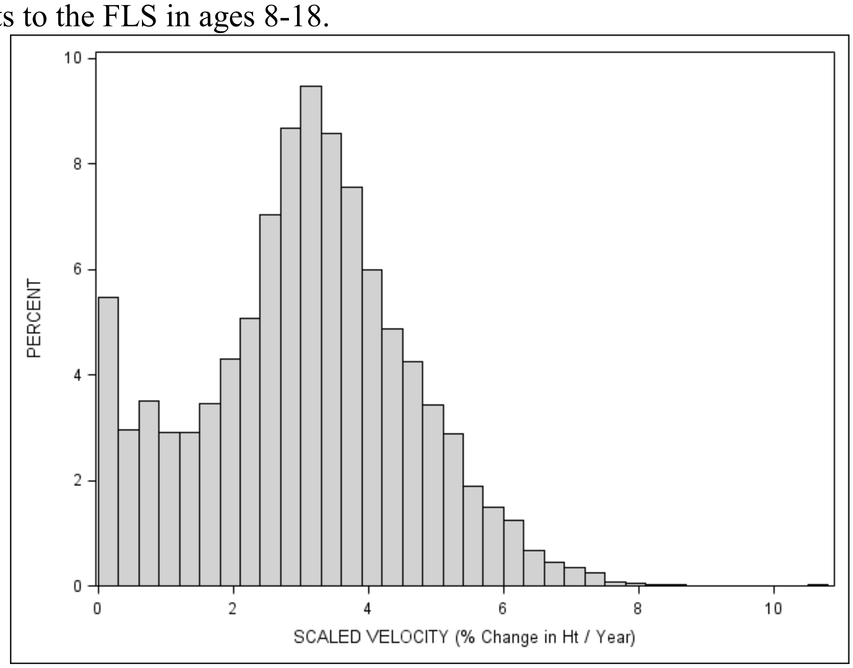


A visual realization of the threshold Δ_4 is shown in graph II of Figure 1 as a horizontal gray line, and the individual Δ_4 values are shown by the interval of vertical lines representing where each person began and ceased growing faster than 4% per year of his or her adulthood height. In this example the individual with the grey lines has a larger Δ_d value because he/she spent more time growing at a rate equal to or above 4% per year. Specifically, the individual with the grey line spends 4.26 years growing faster than 4% per year and the individual with the black line spends 1.55 years. Our study shows that the magnitude of the peak of height velocity is not significantly associated with adulthood BMI; however, Δ_d for some *d* is. Another approach involves an interval of low and high growth thresholds yielding $\Delta_{[a,b]}$ calculated from the height velocity curve. This statistic is the amount of time an individual's growth rate is greater than *a* and less then *b*% per year. Our aim is to find the threshold value, '*d'*, or proper interval, [*a*,*b*], in adolescent height growth rate that best describes an obesity biomarker, BMI, in adulthood.

1.2.2 Data

This section summarizes the distribution of Δ_d and $\Delta_{[a,b]}$; the correlations between Δ_d and $\Delta_{[a,b]}$; and the effects of Δ_d and/or $\Delta_{[a,b]}$ on adulthood BMI. Figure 2 displays the yearly growth rates in height of all 849 individuals from age 8 to 17 years with proper height derivatives calculated from the FLS sample, scaled for their individual adulthood heights.

Figure 2. Histogram showing the scaled first-derivative distribution of all individuals in their adolescent visits to the FLS in ages 8-18.



The histogram represents multiple growth rates representing visits spanning a maximum of ten years per individual, and shows the distribution of the growth rates through adolescence. The horizontal axis represents yearly scaled height growth rates in % of adult height while the vertical axis shows the percentage of a specific growth rate. Ideally, we want to find among these measures in growth rate one that best explains the differences in adult BMI. The delta interval from 2 to 6, for example, appears to coincide with a great majority of the growth rates in the histogram. We reason that the more time someone spends in such a range the longer he/she may be growing, and the lower his/her adulthood BMI may be on average. This also coincides with the convention of a prolonged juvenile state (Sun and Schubert 2009, Sun et al, J Pediatr 2009).

In Table 1, we show summary statistics of the main variables of interest from the FLS in our study (Table 1.A), as well as the distribution of birth years in our FLS sample (Table 1.B). This gives an idea of, in our sample, the values we see for the Δ variables, as well as ages, numbers of visits, and values of the obesity biomarkers.

Table 1. Summary of Data for Chapter 1

A. Summary Statistics					
Time Period	Variable	Mean	S.D.	Min	Max
Adulthood	BMI (kg/m^2)	24.77	4.79	15.00	50.00
	WCI(cm/m%)	53.38	8.22	37.08	98.12
	Age (yr)	34.39	10.33	20.00	60.00
	Female	0.55	0.5	x	x
	# of Visits	6.00	7.27	1	68
Adolescence	Δ_2	1.44	1.82	0	5
	Δ_4	1.16	1.49	0	5
	Δ_6	0.59	0.83	0	4
	$\Delta_{[0,2]}$	0.47	0.79	0	4
	$\Delta_{[0,4]}$	0.75	1.07	0	4
	$\Delta_{[0,6]}$	1.32	1.68	0	6
	$\Delta_{[2,4]}$	0.28	0.46	0	3
	$\Delta_{[2,6]}$	0.84	1.17	0	5
	$\Delta_{[4,6]}$	0.57	0.86	0	5
	# of Visits	7.00	8.17	1	40

B. Birth Year Distn of FLS Sample		
Birth Year	Number	Percent
1930s	683	41.00
1940s	286	17.17
1950s	247	14.83
1960s	200	12
1970s	163	9.78
>1980s	87	5.22
Total	1666	100

(*Note:* Δ unit is years; Subscript of Δ has units percentage of adulthood height)

Table 2 shows the correlations among the delta thresholds and their intervals. The Δ_d represent an individual height growth rate above *d* % per year while $\Delta_{[a,b]}$ corresponds to the growth rate between *a* and *b* % per year of the individual's adulthood height. Consequently, the intervals may describe individuals with steady growth better than a single threshold. In this table there are both intervals and thresholds. The Δ_d also represents an (upper) interval above the individual height growth rate at or above *d* % per year. From Table 2, the more overlapped the two intervals are, the higher their correlation.

Table 2. Correlation Matrix of delta thresholds and intervals

Corr.	Δ_2	Δ_4	Δ_6	$\Delta_{[0,2]}$	$\Delta_{[0,4]}$	$\Delta_{[0,6]}$	$\Delta_{[2,4]}$	$\Delta_{[2,6]}$	$\Delta_{[4,6]}$
Δ_2	1	0.98	0.87	0.57	0.75	0.92	0.76	0.94	0.86
Δ_4		1	0.88	0.57	0.69	0.89	0.62	0.90	0.89
Δ_6			1	0.62	0.71	0.74	0.59	0.65	0.56
$\Delta_{[0,2]}$				1	0.92	0.78	0.41	0.45	0.39
$\Delta_{[0,4]}$					1	0.90	0.74	0.67	0.51
$\Delta_{[0,6]}$						1	0.73	0.91	0.84
$\Delta_{[2,4]}$							1	0.77	0.50
$\Delta_{[2,6]}$								1	0.94
$\Delta_{[4,6]}$	symm.								1

1.2.3 Finding Thresholds

One method we use to find such a threshold is a graphical method of plotting an individual's mean BMI values in adulthood versus his/her Δ_d values in childhood for a range of *d* values. These plots were produced for age intervals 20-30, 30-40, 40-50, and 50-60. Mean BMI was calculated for each individual every ten years. The key idea of this method is to not only find the relationship between Δ_d and individual mean BMI, but also select an appropriate Δ_d most closely associated with BMI in these plots. Graphing $\Delta_{[a,b]}$ against mean BMI in adulthood was used as the visual inspection tool. For example, with growth rates in height we obtain from the FLS the following set of interval statistics:

$$\Delta_{[0,2]}, \Delta_{[2,4]}, \Delta_{[4,6]}, \Delta_6$$

These thresholds and intervals were regressed on adulthood BMI in a two-level Hierarchical Linear Model (HLM2). Multilevel or hierarchical models are appropriate to analyze serial data in which repeated measurements are nested within a child (Raudenbush and Bryk 2002; Goldstein 2003; Sun et al 2008). We investigated the effects on BMI controlling for age decade indicators, indicators of birth year decades (to control secular trend in BMI), and a female indicator. Our aim is to investigate the association between extensive ranges of values for $\Delta_d$and $\Delta_{[a,b]}$ respectively and adulthood obesity outcome; from which we choose one that best explains the variability in the outcome.

To see if time spent growing between *a* and *b* % per year was predictive of adulthood BMI, we studied the effects of candidate intervals $\Delta_{[0,2]}, \Delta_{[2,4]}, \Delta_{[4,6]}, \Delta_6$ – which were intervals that contained all observations in height growth (Figure 2). The interpretation of these interval statistics ($\Delta_{[a,b]}$) is the time (in years) spent growing faster than *a*% and slower than *b%* per year of adulthood height. These intervals, as well as Δ_d for *d* other than *b*, may be tested to select one or more that are highly predictive of adulthood BMI. Likelihood ratio tests along with t-tests for fixed effects will determine a model that best explains the association.

1.2.4 HLM2

Our goal of modeling the longitudinal obesity biomarkers is to infer the impact of adolescent height growth pattern on adulthood health outcomes. To do this we use statistical software package HLM7 (Raudenbush et al. 2011). Occasions, or visits, are lower-level units nested within an individual. This two-level model distinguishes random variation in BMI across time points from that across individuals. Primarily we will look at age ranges 20-30, 30-40, 40-50, and 50-60 in the FLS. The outcome measurements we will be modeling are serial BMI controlling for covariates such as age at examination, gender, race, pregnancies, income, diabetes indicator and smoking status. The covariates in height growth rate will be of interest as they may affect obesity outcomes in adulthood, and their interactions with the gender effect will be tested to see if these growth patterns affect BMI for males and females differently. Further interactions will be tested to elucidate the effect of the height growth variables as it pertains to age groups and secular trends. After BMI analyses are complete, identical models will be fit to waist circumference (cm) divided by height (m). The percentage change of waist circumference with respect to height at each examination will be also studied as a potential obesity indicator, which will be referred to as Waist Circumference Index (WCI).

1.2.5 HLM3

The significance of the secular trend in obesity measurements in adolescence motivates the use of the three-level hierarchical linear model (HLM3) in this study (Sun, Deng, Carrico et al 2011). Adulthood BMI measurements are longitudinal within an individual who is nested within or belongs to a birth year. This will involve studying the effects on adulthood BMI of age groups at the time level (level 1), gender and Δ_d at the individual level (level 2), and secular trend at the birth-year level (level 3).

In particular, the association between Δ_d and obesity outcome, BMI, will be decomposed into individual-level (within-birth-year) and between-birth-year components, β_w and β_b respectively. The β_w explains the expected difference in BMI of two individuals who were born in the same year, but who differ by one unit in Δ_d. The β_b represents the effect of the mean birth-year Δ_d on the mean birth-year BMI at the highest birth year level. The difference, $\beta_c = \beta_b - \beta_w$, defines what is well known in social science and public health applications as the contextual effect of Δ_d (Raudenbush and Bryk 2002; Shin and Raudenbush 2010). It explains the expected difference in BMI between two individuals who have the same Δ_d, but who were born in years that differ by one unit in the mean Δ_d. Popular approaches have been to regress BMI on Δ_d and the sample average birth-year, $\bar{\Delta}_d$. Shin and Raudenbush showed that this approach produces a biased contextual effect while regressing BMI on $\bar{\Delta}_d$, and the empirical Bayes estimate of the population mean Δ_d produces the unbiased contextual effect (Shin and Raudenbush 2010). We estimate the unbiased estimate via the empirical Bayes approach.

With a nontrivial secular trend in growth pattern and obesity outcomes, the contextual effect may be significant. HLM7 produces the posterior residuals given observed data at each level of the fitted model. We perform the model diagnostics based on the posterior residuals.

1.3. Results

1.3.1 BMI Analysis Using HLM2

Our analysis strategy, first, is to select Δ_d and/or $\Delta_{[a,b]}$ that explain variability in adulthood BMI. Let occasion *t* at level one be nested within individual *i* at level two for t=1,…,T_i and i=1,…,N respectively. The two level hierarchical linear model for our analysis is:

Equation 1.

$$BMI_{ti} = \beta_{00} + \beta_{10} * I(Age\ 30-40)_{ti} + \beta_{20} * I(Age\ 40-50)_{ti} + \beta_{30} * I(Age\ 50-60)_{ti}$$

$$+ \beta_{01} * I(Female)_i + \overrightarrow{\beta_{02}} * (\vec{\Delta})_i + \beta_{03} * I(1930s)_i + \beta_{04} * I(1940s)_i +$$

$$\beta_{05} * I(1950s)_i + r_{0i} + e_{ti}$$

Where I(A) is an indicator of event A, $(\beta_{10}, \beta_{20}, \beta_{30})$ are the effects of age groups, β_{01} is the gap in BMI of a female relative to that of a male adult, $\overrightarrow{\beta_{02}}$ is a vector of effects of the height growth, $(\vec{\Delta})_i$ =($\Delta_{[0,2]}, \Delta_{[2,4]}, \Delta_{[4,6]}, \Delta_6$), $(\beta_{03}, \beta_{04}, \beta_{05})$ are the effects of secular trends that indicates birth years in ten-year intervals, $r_{0i} \sim N(0, \tau)$ is a person specific random effect and $e_{ti} \sim N(0, \sigma^2)$ describes random variation across occasions nested within an individual. Because individuals in adolescence differ in the number of visits (Table 1), we may fit $(\vec{\Delta})_i$ without serious multicollinearity. If the individuals had complete data then the full array of $(\vec{\Delta})$ would always add up to a constant. However with the varying visits in the FLS we can properly study these effects without this issue. Stepwise variable selection is used with a significance level of 0.05 as the cutoff. The Δ intervals may be combined as a result of the variable selections. For example two interval effects are subsequent (e.g. [2,4] and [4,6]) and are tested to have the same effect on the outcome, then they can be combined to form one covariate $\Delta_{[2,6]}$. The combining may lead to an easier interpretation of results.

Note that Δ_d represents a length of time an individual grows in height at a rate of d % per year or above. Thus one unit increase in Δ_d implies a year longer at or above the growth rate d. An individual with a high Δ_d has a long period of height growth at or above that rate. The effect of Δ_d on BMI represents the change in BMI as an individual spends one year longer at that growth rate or higher. The implication is that the longer an individual spends growing at d% or higher, the narrower the period of non-trivial growth in height. The fitted models control for the secular trend in BMI by indicators for the decades of birth years, where any birth year after 1960 will be the reference group. Gender will also be taken into account by an indicator variable controlling for the female gender.

The fitted model (Equation 1 with full delta array), reveals effects and the statistical significance of $\vec{\Delta}$, secular trend and gender in Table 3. The female indicator, yearly growth rate of 2% or higher, and the secular trends are all negatively associated with the BMI, ceteris paribus. The variance of 20.39 across individuals indicates that, controlling for the covariates, BMI still varies more than 4cm across individuals on average. Since the variance at level 2 is roughly five times as large as the variance component (4.03) across occasions, it indicates that much more differences in BMI lie across individuals than within after controlling for differences due to gender, secular trends, height growth patterns, and age.

We see that individuals who have more years in adolescence growing less than 2% per year is indicative of significantly higher BMI. The time spent between 2 and 4%, the time spent between 4 and 6%, and the time spent greater than 6% affect BMI in the same positive direction, and are all significant, so in the interest of interpretability will be combined to form Δ_2 in the final model (LRT X^2=4.18 , p-value=0.120) to produce the result in Table 4. This LRT shows that dropping terms from the initial model does not result in a model that explains significantly less information about BMI.

Table 3 shows that the adolescent height growth pattern is significantly associated with adulthood BMI, controlling for significant effects of age, gender, and secular trend. The results show that the amount of time an individual spends at a high growth rate Δ_2 is predictive of lower adulthood BMI. That is, the effect of Δ_2 is negative, whi ch implies that a person spending a comparatively long period of 2% or higher growth in height will have a relatively narrower period of height growth. The narrower period of rapid height growth leads to comparatively lower BMI in adulthood. Even more telling is the fact that an extended period of not growing, or growing at a low rate $\Delta_{[0,2]}$ is indicative of high adulthood BMI. The implication is that the person that does not spend an extensive period of prolonged growth in height, will experience comparatively higher BMI in adulthood.

Table 3. HLM2 (Equation 1 Reduced):

Fixed Effect	Estimate	S.E.	t-ratio	d.f.	p-value
Intercept	26.33	0.36	73.43	1185	<0.001
Female	-1.60	0.31	-5.19	1185	<0.001
I(1930s)	-2.31	0.41	-5.67	1185	<0.001
I(1940s)	-2.03	0.38	-5.42	1185	<0.001
I(1950s)	-1.62	0.38	-4.26	1185	<0.001
$\Delta_{[0,2]}$	0.66	0.20	3.29	1185	0.001
Δ_2	-0.50	0.09	-5.33	1185	<0.001
Age 30-40	1.82	0.08	22.46	4492	<0.001
Age 40-50	3.34	0.09	35.54	4492	<0.001

Age 50-60	4.71	0.11	44.81	4492	<0.001

Random Effect	S.D.	Var Comp	d.f.	X^2	p-value
Level-2	4.52	20.39	1185.00	23565.28	<0.001
Level-1	2.01	4.03			

In the two-level analysis in Table 3 there are 2 contrasting effects on BMI of adolescent height growth pattern, as well as significant effects of age, gender and secular trend effects. The Δ result shows that an extended period of time that an individual spends growing at a high rate in adolescence lowers adulthood BMI, but an extended time of low rate growth may be detrimental, on average. To put this in perspective; there is a 10-year period that individuals can spend growing in childhood, which means that an individual cannot have high values for both $\Delta_{[0,2]}$ and Δ_2 at the same time. Rather most individuals in the FLS have either moderate values for each or extreme values of just one of them. This phenomenon is shown more precisely in the three-level analysis.

1.3.2 BMI Analysis Using HLM3

Demonstrated previously, the two-level analysis there is a very significant secular trend in adulthood BMI for the Fels data. This result prompted the use of a three-level hierarchical data analysis. This method involves studying longitudinal time measurements on individuals who belong to specific birth years as recorded in the FLS. The three-level model will be fit to longitudinal BMI measurements with age covariates at level one, gender and childhood height growth patterns at level 2 or the individual level, and the contextual effect of adolescent height growth rate at level 3, or birth-year level. This model is shown as equation 2.

Equation 2.

$$BMI_{tij} = \beta_{00} + \beta_{10} * I(Age\ 30 - 40)_{tij} + \beta_{20} * I(Age\ 40 - 50)_{tij} + \beta_{30} * I(Age\ 50 - 60)_{tij}$$

$$+ \beta_{01} * I(Female)_{ij} + \overrightarrow{\beta_{02}} * (\vec{\Delta})_i + \beta_{001} * (\mu_{\Delta})_j + \gamma_{00j} + r_{0ij} + e_{tij}$$

For examinations t, on individuals i that belong to birth year j, we look at the effect of $\Delta_{[0,2]}$ and Δ_2 at the individual level (level 2) and their contextual effects (at level 3). Recall the birth year and between birth-year effects β_w and β_b of a covariate X. When a covariate X has a non-zero contextual effect $\beta_c = \beta_b - \beta_w$ on BMI, but is modeled to produce a single effect, β, on the outcome, the single effect of X may be shown to be $\beta=\rho^* \beta_b-(1-\rho) \beta_w$ where ρ is the intra-cluster correlation coefficient (Raudenbush and Bryk 2002). For X= $\Delta_{[0,2]}$, the β is a weighted average of the within-birth-year association β_w and the between-birth-year association β_b with BMI where the weight is ρ. To represent precisely the within-group association β_w that is different from the between-group association β_b, we fit both β_w and β_c. Pervasive analysis of such a contextual effect in the literature is to use X and the sample group-mean $\bar{X}$ as the two covariates. Shin and Raudenbush (2010) showed that this approach produces not only biased β_c but biased effects of other group-level covariates. They showed that use of X and the empirical Bayes estimate of the population birth year mean of X as two covariates would produce unbiased estimates of β_c, the approach we use in the HLM3 below. Because the birth year means of $\Delta_{[0,2]}$ and Δ_2 are highly correlated, we include only one contextual effect (the one with stronger association) shown in Equation 2.

Table 4. Results of HLM 3 Model for BMI

Fixed Effect	Estimate	S.E.	t-ratio	d.f.	p-value
Intercept	15.45	1.82	8.47	52	<0.001
$\Delta_{[0,2]}$ (contextual)	6.98	1.43	4.88	52	<0.001
Female	-2.55	0.48	-5.33	389	<0.001
$\Delta_{[0,2]}$ (within)	1.06	0.31	3.41	389	<0.001
Δ_2 (within)	-0.44	0.18	-2.47	389	0.014
Age 30-40	1.93	0.20	9.85	2517	<0.001
Age 40-50	3.40	0.25	13.40	2517	<0.001
Age 50-60	4.99	0.24	20.77	2517	<0.001

Random Effect	S.D.	Var Comp	d.f.	X^2	p-value
Level-1	2.04	4.15			

Level-2	3.99	15.90	389	8385.57	<0.001
Level-3	0.09	0.01	52	55.89	0.331

The results are shown in Table 4, where we see that the gender, age and Δ_2 effects are consistent with the findings in the two-level models, in direction and significance. We see that $\Delta_{[0, 2]}$ is positively associated with adult BMI at individual level (1.06) as well as at secular-trend level or birth-year level (6.98). The effect 1.06 at the individual level represents the expected difference in adulthood BMI between two individuals who were born in the same year, but who differ by one unit in their $\Delta_{[0, 2]}$. The contextual effect 6.98 is the expected difference in adulthood BMI between two individuals who have the same $\Delta_{[0, 2]}$ but who were born in birth years that differ by one unit in the average age group value of $\Delta_{[0,2]}$. Therefore, the secular trend in the effect of adolescent height growth rate on adulthood BMI is statistically significant, so that birth year differences in mean height growth impact adulthood BMI.

1.3.3 Waist Circumference Analysis

To offer another perspective on the effect of the Δ variables, and height growth in adolescence on adulthood obesity we repeat the analysis conducted using BMI on individual waist circumference measurements divided by their corresponding longitudinal height measurements. For better interpretation we multiply these values by 100 to have percentage changes in this measurement relative to height measurement. This is referred to as the waist circumference index (WCI). The two-level results are found in Table 5.

Table 5. Two-level WCI analysis Results

Fixed Effect	Estimate	S.E.	t-ratio	d.f.	p-value
Intercept	51.00	0.59	85.93	956	<0.001
Female	-1.35	0.55	-2.44	956	0.015
I(1930s)	-4.88	0.83	-5.89	956	<0.001
I(1940s)	-4.50	0.68	-6.61	956	<0.001
I(1950s)	-2.40	0.62	-3.85	956	<0.001
$\Delta_{[0,2]}$	1.51	0.34	4.42	956	<0.001
Δ_2	-0.61	0.16	-3.86	956	<0.001
Age 30-40	4.88	0.18	26.53	8713	<0.001
Age 40-50	8.06	0.21	38.34	8713	<0.001
Age 50-60	11.56	0.25	46.42	8713	<0.001

Random Effect	S.D.	Var Comp	d.f.	X^2	p-value
Level-2	7.13	50.87	956.00	16108.87	<0.001
Level-1	3.34	11.16			

The age, secular trend, gender, and Δ effects are all comparable to the effects in the two-level BMI analysis in signs and significance. Values may differ based on the scale of the outcome variables. That is, there is a positive age trend, significant secular trend showing WCI has been increasing, and that women generally have lower WCI. Also the Δ effects very much mimic the effects seen in BMI. The significant secular trend effect in Table 5 again leads us to study the effect of Δ at generational level, and control for generational variability in WCI. Results for this analysis are shown similarly to that of the BMI analysis, in Table 6. Again only the contextual effect of $\Delta_{[0,2]}$ was included in order to maintain the consistency of reported effects across analyses.

Table 6. Three-Level WCI Analysis Results

Fixed Effect	Estimate	S.E.	t-ratio	d.f.	p-value
Intercept	30.08	3.47	8.67	52	<0.001
$\Delta_{[0,2]}$ (contextual)	13.82	2.61	5.29	52	<0.001
Female	-2.37	0.86	-2.76	389	0.006
$\Delta_{[0,2]}$ (within)	1.80	0.49	3.67	389	<0.001
Δ_2 (within)	-0.66	0.31	-2.12	389	0.035
Age 30-40	5.19	0.24	21.42	2518	<0.001
Age 40-50	8.27	0.27	30.53	2518	<0.001
Age 50-60	12.12	0.32	37.67	2518	<0.001

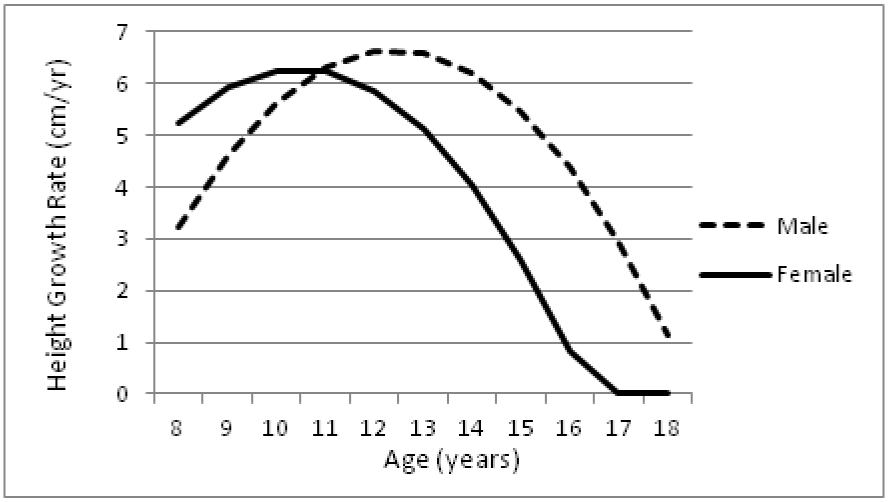
Random Effect	S.D.	Var Comp	d.f.	X^2	p-value
Level-1	3.50	12.28			
Level-2	6.34	40.22	346	5323.93	<0.001
Level-3	0.74	0.55	52	65.05	0.106

The three level analyses for WCI are consistent with those of BMI, with the Δ values having the same impact on WCI that they had on BMI in signs and statistical significance. The WCI analysis gives an alternative look at the effect of adolescent height growth on another obesity biomarker.

1.3.4 Prediction Models with Δ

Since the Δ variables are found to be predictive of adulthood BMI and WCI, it may be of use to predict one of these biomarkers using individual adolescent height growth data. The Δ variables may be calculated directly from the data during regularly scheduled examinations using individual height measurements. During typical examinations these are plotted against national growth charts (such as Tanner and Whitehouse 1976). Individual height derivative curves can be calculated likewise, and, then, extraction of the amount of time they spent growing at specific rates can be ascertained. In the FLS we can find the height growth patterns of adolescent boys and girls against their ages by fitting a two-level model of repeated observations on individuals in adolescence. The pattern found in the FLS in Figure 3 shows the expected phenomenon of earlier growth spurts for adolescent females than males, and typical height growth values in adolescence.

Figure 3. Mean Height Growth Rate values in FLS



In the main analysis described in this paper, the Δ values are scaled for adulthood height. This makes characterizing them as an obesity biomarker more interpretable as a percentage change in eventual height since individuals all can reach different adulthood heights. In use one may be interested in using an individual's current height growth for forecasting their adulthood BMI (or WCI). It is noted that not all individuals would fit well to the average height growth curves, but they are at least a stable projection. For forecasting purposes, we fit an HLM-2 using Δ values not scaled by adulthood height in the FLS these results may be found in Table 7.

Table 7. HLM-2 results for BMI using Δ not scaled by adulthood height

Fixed Effect	Coefficient	s.e.	*t-ratio*	*d.f.*	p-value
Intercept	26.35	0.36	72.83	1185	<0.001
Female	-1.62	0.31	-5.24	1185	<0.001
I(1930s)	-2.32	0.41	-5.68	1185	<0.001
I(1940s)	-2.04	0.37	-5.43	1185	<0.001
I(1950s)	-1.62	0.38	-4.25	1185	<0.001
$\Delta_{[0,2]}$	0.37	0.12	3.05	1185	0.002
Δ_2	-0.28	0.05	-5.28	1185	<0.001
Age 30-40	1.82	0.08	22.45	4492	<0.001
Age 40-50	3.34	0.09	35.54	4492	<0.001
Age 50-60	4.71	0.11	44.80	4492	<0.001

Random Effect	S.D.	Estimate	*d.f.*	χ^2	*p*-value
Level-2	4.52	20.40	1185	23588.1	<0.001
Level-1	2.01	4.03			

Using the same method described previously for studying the Δ intervals and thresholds, we arrive at a model that includes $\Delta_{[0,2]}$ and Δ_2, the two values that measure the amount of time someone spends growing between 0 and 2 cm per year and the amount of time someone spends growing faster than 2 cm per year, respectively. Thus an individual could use their own height data to predict adulthood BMI via the model in Table 7. Similarly, the resulting model for WCI is in Table 8. For example (predicting WCI or BMI), a female at the age of 18 could predict her BMI or WCI in her 20s to be the sum of the intercept, the female indicator slope parameter, and her two Δ values multiplied by their corresponding slope parameters. An individual with this information could make behavior lifestyle changes based on these findings, and whether or not they have a higher propensity for adulthood obesity based on their adolescent height growth profile. For example, a female with $\Delta_{[0,2]}=2$ and $\Delta_2=8$, predicted BMI in her 20s would be 23.15 (95% CI [10.4, 35.9]) whereas a female with $\Delta_{[0,2]}=8$ and $\Delta_2=2$, predicted BMI in her 20s would be 27.13 (95% CI [14.3, 39.9]). The confidence intervals are calculated using a normal distribution with mean described above the combined variances of both levels for total variance.

Table 8. HLM-2 Results for WCI using Δ not scaled by adulthood Height

Fixed Effect	Coefficient	s.e.	*t-ratio*	*d.f.*	p-value
Intercept	51.07	0.60	85.13	956	<0.001
Female	-1.36	0.56	-2.44	956	0.015
I(1930s)	-4.91	0.83	-5.92	956	<0.001
I(1940s)	-4.52	0.68	-6.64	956	<0.001
I(1950s)	-2.41	0.62	-3.86	956	<0.001
$\Delta_{[0,2]}$	0.86	0.20	4.22	956	<0.001
Δ_2	-0.35	0.09	-3.95	956	<0.001
Age 30-40	4.88	0.18	26.51	8713	<0.001
Age 40-50	8.06	0.21	38.32	8713	<0.001
Age 50-60	11.55	0.25	46.40	8713	<0.001

Random Effect	S.D.	Estimate	*d.f.*	χ^2	*p*-value
Level-2	7.13	50.90	956	16105.12	<0.001
Level-1	3.34	11.17			

1.3.5 Simulation Confirmation

The Empirical Bayes estimation of a contextual effect has been shown to produce an unbiased estimate in a two-level case (Shin and Raudenbush 2010). Here, however we have the contextual effect at level three and the within effect at level two. To the unbiased estimation of the contextual effect in the three-level setting, we simulate a data set with the same structure as our FLS sample (as in the number of visits per individual per birth year) , and compare estimation of the contextual effect using sample mean versus the EB mean. The results are shown in Table 9.

The aim values were from an HLM3 similar to the models shown before. The difference is that these were conducted with only one level-2 Δ or X predictor . With perfect data the two Δ are perfectly correlated and one would suffice. As noted, the structure of the FLS data allowed us to simulataneously study the effects of the entire range of $\Delta_{[a,b]}$. The simulated values are shown in the “True Effects” on the last column of the table.

Table 9. Results of Simulated three-level data

Sample Mean Model			Empirical Bayes Model			
Effect	Estimate	S.E.	Effect	Estimate	S.E.	True Effects
Intercept	9.51	0.64	Intercept	7.83	0.88	7.06
Sample Mean of X	4.93	0.83	EB Mean of X	7.35	1.24	8.27
Female	-2.12	0.38	Female	-2.12	0.38	-2.20
X	0.91	0.25	X	0.91	0.25	1.04
Age 30-40	2.10	0.10	Age 30-40	2.10	0.10	1.93
Age 40-50	4.18	0.12	Age 40-50	4.18	0.12	3.40
Age 50-60	5.02	0.13	Age 50-60	5.02	0.13	5.00

We see that the within-group effects of X remain the same in both models, as analogous to the two-level analysis of Shin and Raudenbush (2010). The contextual effect of the sample mean of X (4.93) underestimates the true contextual effect, whereas the effect of the EB mean of X (7.35) is much closer to the true effect 8.27. Though this is one sample, it lends confidence that the proven result from Shin and Raudenbush holds when the effects are at levels two and three instead of levels one and two.

1.4 Discussion

Studying height velocity in the manner presented in this paper allows an individual's entire growth profile to be used for the study and analysis of adulthood obesity. Our method takes into account all adolescent periods of height growth. Different aspects of the height velocity curve seem to be predictive of adulthood obesity markers. Elongated time where an individual is not growing is shown here to be predictive of adult obesity (higher BMI, WCI), and elongated time of high growth rate is shown to be associated with lower adulthood obesity (lower BMI, WCI). The magnitude of the velocity is not found to be significant, but rather the length of time the individual is continuously growing in adolescence.

The analysis reveals several findings and also raises questions on the subject matter. The findings reveal the highly significant secular trend in the FLS data as well as the age trend of BMI and WCI in adulthood. That is, the longer a child spends growing above 2% of his or her adult height per year, the lower their BMI is likely to be in adulthood. Another way of looking at this is that if an individual is not growing in height, but the weight is increasing, then it must be that BMI increases. It has also been shown here that the amount of time spent growing in the two Δ rate brackets is predictive of adult BMI and WCI. That is, the longer an individual spends growing at a high rate, the lower his or her BMI or WCI in adulthood is on average, and the longer they spend growing at a low rate, the higher their adulthood BMI or WCI is on average.

Characteristics of individuals with different Δ values include either a short growth spurt or a continued period of slow growth. An individual with a high $\Delta_{[2,+]}$ value would be hypothesized to have a long period of relatively high growth that might or might not have a very tall peak height velocity. But the overall duration of height growth is relatively narrow, compared to individuals with a low Δ_2. Individuals with high $\Delta_{[0,2]}$ values are the individuals who either never grow at a high rate, or who briefly grow at a high rate and then return to slow growth. Consequently, these individuals have a prolonged height growth over an extensively long period to reach adulthood height, and never grow at a high rate. The two values, $\Delta_{[0,2]}$ and Δ_2, describe the features of a second-derivative (height acceleration) curve in this way.

The peak height velocity was not shown to be a significant predictor of adulthood obesity in this sample; rather the length of time that encompasses peak height velocity is relevant as our study shows. This finding is independent of the timing of pubertal events but is instead a summary of duration of height growth throughout adolescence. The timing of pubertal growth spurts has been shown previously to impact adulthood growth patterns as mentioned previously, but this finding augments those findings. Importantly, the two Δ measurements found to be significant on adulthood obesity biomarkers explain the entirety of the individual adolescent height growth profile. As an individual has growth spurts in adolescence, the length of these periods of growth does, in fact, affect the likelihood for obesity in adulthood.

Chapter 2

Simulation Study of Contextual Effects in Non-linear Hierarchical Linear Models (HGLM)

2.1. Simulation Setup and Explanation

2.1.1 Background

The goal of this simulation is to study the contextual effects of a continuous predictor on non-normal outcomes in a generalized linear model framework. We will simulate a continuous predictor variable, *X*, and with the simulated values we will know the true cluster-level mean of X. We simulate N_j units within cluster j for j=1,...,1000 (1000 groups for large-sample estimation stability). We then will be able to compare fitting a model with the true cluster mean of X with the sample mean of the upper level units and the Empirical Bayes estimate of the upper level unit's mean.

We simulate the model based upon the true means, using the appropriate inverse link function to obtain an outcome with the specified distribution. To be more complete, the idea of a generalized linear model will be introduced. The model written as Y=XB+E, E must be normally distributed, and in a hierarchical linear model, must be independent across units. In a normal linear regression model, an outcome Y is modeled as a function of a vector, B, of slope parameters and an intercept and a data matrix, X, with random error, E, normally distributed. These parameters B are estimated to show the outcome variable Y as a linear function of explanatory variables. The goal of a regression analysis is to obtain estimates of the elements of B, and to quantify the amount of error remaining in E. Diagnostics are performed in a traditional analysis to ensure that the assumptions placed on the modeling analysis are verified once the estimation is complete.

The normality assumption on the errors in the estimation is not correct, however, when the data takes on such forms as binary or count data. The error terms E will not satisfy all of the assumptions on the model. In this case, the theory of hierarchical generalized linear models (HGLM) is implemented. The idea of the HGLM is to relate the mean (μ) with the outcome (Y) through a non-linear link function $g(\cdot)$. This link function follows from the theory of exponential family of distributions from which the forms of these link functions are found. What follows from this theoretical framework is that one can now relate the Y to its mean, through the link function $g(\cdot)$. It then has the form $g(Y)= \mu$, or $g(Y)=XB+E$ as seen before. Non-normal outcomes that we will focus on in our studies are all of the exponential family of distributions, thus can follow the theoretical framework of generalized linear models. This allows virtually the same goal of a regression analysis, where we estimate parameters showing relationships of explanatory variables to the outcome, but our outcomes will not be normally distributed continuous data.

In the HGLM setting it is also interesting to quantify the error terms at specific levels of the analysis, where we can interpret variance amounts by estimating variance components. In the case of hierarchical data, it is assumed the errors are independent and normally distributed at each level. This type of data occurs when you have measurement units that have an inherent correlation structure to them, because of a grouping variable. That is the data are correlated within nested units, but not across them. Examples of hierarchical data are students within schools, longitudinal measurements on individuals, or patients within hospitals. In these cases there is variation in the outcome measurement due to both levels. This is called nested or hierarchical data, and the HLM framework is applied to HGLM when the link functions are used as described above.

The estimation of contextual effects in this setting has yet to be established. Software commonly used for implementation of HGLM models is SAS Proc NLMIXED, SAS Proc GLIMMIX, GLLAMM package in STATA (Rabe-Hasketh 2004), as well as use of the HLM software (Raudenbush, Bryk, Congdon). The models shown in this simulation section are fit using SAS Proc GLIMMIX, but are periodically checked for consistency with other software. The reason SAS Proc GLIMMIX can be used is because of the ability to use Gaussian Quadrature as the method of integration for maximizing the likelihoods of these models (Rabe Hesketh, Skrondal 2001). Integration of random effects is carried out with numerical approximations, and Gaussian Quadrature has been shown to be very accurate in these settings, which is why it is used by the software packages.

2.1.2 Set Up

The first simulation will use binary outcome data that is distributed in a Binomial distribution. The corresponding link to the Binomial distribution is a logit link. More specifically, this simulation goes as follows.

1) Simulate $X_{ij} = \mu_{X_j} + e_{X_j}$, where $e_{X_j} \sim N(0, {\sigma_{xx}}^2 \equiv 1)$ and $\mu_{X_j} = r_X + u_{X_j}$ for $u_{X_j} \sim N(0, \tau_{XX})$

2) Simulate p_{ij} such that

$$Logit\left(p_{ij}\right) = \log\left(\frac{p_{ij}}{1-p_{ij}}\right) = \beta_0 + \beta_1 * \left(X_{ij} - \mu_{X_j}\right) + \beta_2 * \mu_{X_j} + u_j,$$

$$\text{or, } p_{ij} = \frac{\exp\left(\beta_0 + \beta_1 * \left(X_{ij} - \mu_{X_j}\right) + \beta_2 * \mu_{X_j} + u_j\right)}{1 + \exp\left(\beta_0 + \beta_1 * \left(X_{ij} - \mu_{X_j}\right) + \beta_2 * \mu_{X_j} + u_j\right)}$$

Where parameters for the simulation are set such that: $\beta_0, \beta_1 = 1$ and $\beta_2 = 2$, and $u_j \sim N(0, \Psi)$ for a series of values for Ψ.

3) Simulate the binary outcome, $Y_{ij} = Binomial(p_{ij})$ for level-1 variability.

4) Fit three models (superscripts on β denotes model):

$$\text{Model A: } Logit\left(Y_{ij}\right) = \beta_0 + \beta_1 * \left(X_{ij} - \mu_{X_j}\right) + \beta_2 * \mu_{X_j} + u_j$$

$$\text{Model B: } Logit\left(Y_{ij}\right) = \beta_0{}^B + \beta_1{}^B * \left(X_{ij} - \bar{X}_j\right) + \beta_2{}^B * \bar{X}_j + u_j$$

$$\text{Model C: } Logit\left(Y_{ij}\right) = \beta_0{}^C + \beta_1{}^C * \left(X_{ij} - \mu_{X_j}^{EB}\right) + \beta_2{}^C * \mu_{X_j}^{EB} + u_j$$

Where $\bar{X}_j = \sum_{i=1}^{Nj} X_{ij}$ is the sample mean of X_{ij} for cluster j. In the same way $\mu_{X_j}^{EB}$ is the Empirical Bayes estimate of μ_{X_j}, as defined by Shin and Raudenbush (2010). This is defined as a convex function:

$$\mu_{X_j}^{EB} = \delta_j * \bar{X}_j + \left(1 - \delta_j\right) * \hat{r}_X$$

Where $\widehat{\delta_j} = \frac{\widehat{\tau_{XX}}}{\widehat{\tau_{XX}} + \frac{\widehat{\sigma_{xx}{}^2}}{Nj}} \in [0,1]$ is the reliability of the sample mean $\bar{X}_j$ as an estimate for μ_{X_j}; and $\hat{r}_X$ is the estimated mean (intercept) of X from a simple two-level model for X, from which δ_j is also calculated (Raudenbush&Bryk 1992, chap.3). The terms $\widehat{\tau_{XX}}$ and $\widehat{\sigma_{xx}{}^2}$ are the estimates of the lower and upper level variances in X. This represents the fact that X varies across level-1 units as well as level-2 units. The higher the reliability ($\delta_j \rightarrow 1$), the better the sample means are as estimates for the true means, and the lower the reliability ($\delta_j \rightarrow 0$), the less each group's sample mean is representative of the true group mean. As the convex function shows, the more reliable the means $\bar{X}_j$, the more weighted the Empirical Bayes estimate is weighed by the sample mean. As noted by Shin and Raudenbush (2010), the reliability of the sample mean approaches 1 as N_j gets large.

The true group mean μ_{X_j} of X_{ij} is simulated from step 1. Throughout the simulations and fitting of these models, β_0 is the intercept term, but β_1 will also be referred to as the within effect of X, and β_2 the between effect of X. The difference between the within effect and the between effect is known as the contextual effect of X_{ij}. The vectors $\vec{\beta}^B = (\beta_0{}^B, \beta_1{}^B, \beta_2{}^B)$ and $\vec{\beta}^C = (\beta_0{}^C, \beta_1{}^C, \beta_2{}^C)$ are estimates of the true β_0, β_2 and β_3, corresponding to models B and C.

An illustration of the within, between and contextual effects is helpful for the understanding of the contextual effects concept, and is well portrayed by Raudenbush and Bryk in the Hierarchical Linear Models text book (pg140, 2002). The within effect is interpreted as the difference in the outcome (through the link function) from the same upper level unit that differ in X_{ij} by 1. The between effect is the difference in the outcome of two upper level groups where μ_{X_j} are 1 unit different from each other. The contextual effect is then the difference of two individuals who have the same X_{ij} values but belong to upper level units that differ by 1 unit in μ_{X_j}.

According to the results for a normal linear contextual effect model (Shin and Raudenbush 2010) models B and C should have the same estimates of β_1, but β_0 and β_2 are unbiased estimates of the true values in model C, whereas their estimates are biased in model B. Estimates of Ψ are known to be biased in both models B and C. This will be studied in the HGLM framework to see if the same theory is shown true in the models using link functions to the non-Normally distributed outcome.

We want to see the impact of the three different representations of the group means of X_{ij} on the three different model estimates. We will compare estimates of β_0, β_1, β_2, and Ψ to assess the impact of using the different group means. We will complete this simulation 75 times to cover the combinations of the following simulation parameters:

- τ_{XX}=(0.1, 0.2, 0.3, 0.4, 08)
- N_j = (5, 10, 20, 40, 80)
- Ψ=(0.2, 0.5, 1.0)

This entire procedure is repeated three times, with binary data with a logit link, binary data with a probit link function, and Poisson data with a log link function. The goal of this simulation study is to examine the bias in estimation of the contextual effect via the generalized linear models A, B, and C and to compare the results to those of the normal linear models described in Shin and Raudenbush (2010). Shin and Raudenbush showed how to correct the bias of estimating the contextual effects in normal models, and we want to see if the bias correction applies in the generalized linear model setting with various outcome types and distributions. It is prudent to show the steps of simulation with each of the outcome distributions we use, as the model expressions are slightly different form based on the link function to the response.

For the binary data with a probit link, steps 2, 3, and 4 are as follows:

2) Simulate p_{ij} such that $\Phi^{-1}(p_{ij}) = \beta_0 + \beta_1 * \left(X_{ij} - \mu_{X_j}\right) + \beta_2 * \mu_{X_j} + u_j$, or,

$$p_{ij} = \Phi(\beta_0 + \beta_1 * \left(X_{ij} - \mu_{X_j}\right) + \beta_2 * \mu_{X_j} + u_j)$$

where parameters for the simulation are set such that: $\beta_0, \beta_1 = 1$ and $\beta_2 = 2$, and $u_j \sim N(0, \Psi)$.

3) Simulate the binary outcome, $Y_{ij} = Binomial(p_{ij})$, so that the level-1 variability is that of a Binomial distribution with specified probability.

4) Fit three models:

Model A: $\Phi^{-1}(p_{ij}) = \beta_0 + \beta_1 * \left(X_{ij} - \mu_{X_j}\right) + \beta_2 * \mu_{X_j} + u_j$

Model B: $\Phi^{-1}(p_{ij}) = {\beta_0}^B + {\beta_1}^B * \left(X_{ij} - \bar{X}_j\right) + {\beta_2}^B * \bar{X}_j + u_j$

Model C: $\Phi^{-1}(p_{ij}) = {\beta_0}^C + {\beta_1}^C * \left(X_{ij} - \mu^{EB}_{X_j}\right) + {\beta_2}^C * \mu^{EB}_{X_j} + u_j$

Where the function, Φ^{-1}, is the inverse of the standard normal cumulative density function.

Then, for the Poisson data with a log link (a log-linear model), steps 2, 3 and 4 are as follows:

2) Simulate λ_{ij} such that $ln(\lambda_{ij}) = \beta_0 + \beta_1 * \left(X_{ij} - \mu_{X_j}\right) + \beta_2 * \mu_{X_j} + u_j$, or

$$\lambda_{ij} = \exp\left(\beta_0 + \beta_1 * \left(X_{ij} - \mu_{X_j}\right) + \beta_2 * \mu_{X_j} + u_j\right)$$

where parameters for the simulation are set such that: $\beta_0, \beta_1 = 1$ and $\beta_2 = 2$, and $u_j \sim N(0, \Psi)$.

3) Simulate the Poisson distributed outcome, $Y_{ij} = Poisson(\lambda_{ij})$, so that the level-1 variability is that of a Poisson random variable in which the variance is equal to the mean.

4) Fit three models:

Model A: $ln\left(\lambda_{ij}\right) = \beta_0 + \beta_1 * \left(X_{ij} - \mu_{X_j}\right) + \beta_2 * \mu_{X_j} + u_j$

Model B: $ln\left(\lambda_{ij}\right) = \beta_0{}^B + \beta_1{}^B * \left(X_{ij} - \bar{X}_j\right) + \beta_2{}^B * \bar{X}_j + u_j$

Model C: $ln\left(\lambda_{ij}\right) = \beta_0{}^C + \beta_1{}^C * \left(X_{ij} - \mu^{EB}_{X_j}\right) + \beta_2{}^C * \mu^{EB}_{X_j} + u_j$

Estimates for β_0, β_1, β_2, and Ψ will be tabulated for all of the above outlined scenarios, then studied for the behavior of the contextual effect in the HGLM framework.

2.2 Results for Logit, Probit and Poisson

When the group size increases, bias decreases and efficiency increases by all models. Overall, when the sample size in the upper level unit increases, the sample mean's reliability approaches one, so the results using the EB estimated mean and the sample mean are similar. As the cluster-level variance of X (τ_{XX}) gets small compared to Ψ and σ^2_{xx}, the results begin to become less accurate in the estimates of the parameters, and the increasing standard errors of the estimates. For $\tau_{XX} = 0.1$, in most cases $N_j < 40$, the reliability of the sample means become quite small which in turn gives the estimates of the contextual effects less precision. The large standard errors of the effect estimates in these cases lead conservative hypothesis tests that fail to reject the null hypothesis often as well as unintuitive estimates of the contextual effect itself. Tables 10-12 show the results for the β_2 estimates in the three models. Tables will be shown with only β_2 estimates since this is the part that deals with the contextual effect; other terms resulting from the estimation are contained in the appendix.

Table 10. Logistic Regression Simulation β_2 results for models A, B and C and Ψ=0.2

Model A				Model B				Model C			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
2.51	0.35	0.1	5	1.05	0.08	0.1	5	2.17	1.08	0.1	5
1.30	0.17	0.2	5	1.05	0.07	0.2	5	1.78	0.06	0.2	5
1.79	0.12	0.3	5	1.22	0.07	0.3	5	2.11	0.05	0.3	5
1.84	0.10	0.4	5	1.33	0.07	0.4	5	1.77	0.15	0.4	5
1.95	0.07	0.8	5	1.71	0.06	0.8	5	1.96	0.07	0.8	5
2.23	0.28	0.1	10	1.14	0.09	0.1	10	3.28	1.19	0.1	10
2.06	0.14	0.2	10	1.38	0.08	0.2	10	1.86	0.04	0.2	10
1.90	0.09	0.3	10	1.42	0.07	0.3	10	1.81	0.02	0.3	10
1.95	0.07	0.4	10	1.60	0.06	0.4	10	2.00	0.09	0.4	10
1.92	0.05	0.8	10	1.75	0.04	0.8	10	1.88	0.05	0.8	10
1.88	0.21	0.1	20	1.06	0.09	0.1	20	1.54	1.03	0.1	20
2.00	0.11	0.2	20	1.46	0.08	0.2	20	1.80	0.03	0.2	20
2.05	0.07	0.3	20	1.69	0.06	0.3	20	1.97	0.01	0.3	20
1.86	0.06	0.4	20	1.61	0.05	0.4	20	1.82	0.07	0.4	20
1.88	0.04	0.8	20	1.80	0.04	0.8	20	1.86	0.04	0.8	20
2.00	0.18	0.1	40	1.16	0.10	0.1	40	1.97	0.04	0.1	40
2.09	0.09	0.2	40	1.73	0.07	0.2	40	2.00	0.01	0.2	40
2.01	0.06	0.3	40	1.78	0.06	0.3	40	1.95	0.01	0.3	40
2.00	0.05	0.4	40	1.88	0.05	0.4	40	1.95	0.01	0.4	40
1.97	0.03	0.8	40	1.95	0.03	0.8	40	1.95	0.00	0.8	40
2.05	0.16	0.1	80	1.36	0.11	0.1	80	1.81	0.02	0.1	80
1.93	0.08	0.2	80	1.73	0.07	0.2	80	1.92	0.01	0.2	80
2.03	0.06	0.3	80	1.90	0.06	0.3	80	2.00	0.01	0.3	80
1.93	0.04	0.4	80	1.85	0.04	0.4	80	1.87	0.00	0.4	80
1.93	0.02	0.8	80	1.90	0.02	0.8	80	1.89	0.00	0.8	80

Table 11. Logistic Regression Simulation β_2 results for models A, B and C and Ψ=0.5

Model A				Model B				Model C			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
2.28	0.38	0.1	5	0.97	0.09	0.1	5	7.58	4.79	0.1	5
1.76	0.20	0.2	5	1.11	0.08	0.2	5	1.33	0.12	0.2	5
1.78	0.13	0.3	5	1.14	0.08	0.3	5	1.72	0.07	0.3	5

1.95	0.11	0.4	5	1.39	0.07	0.4	5	1.91	0.16	0.4	5
1.88	0.07	0.8	5	1.62	0.06	0.8	5	1.85	0.08	0.8	5
1.63	0.32	0.1	10	0.96	0.09	0.1	10	1.16	0.78	0.1	10
1.68	0.16	0.2	10	1.12	0.09	0.2	10	1.75	0.07	0.2	10
1.87	0.11	0.3	10	1.44	0.08	0.3	10	1.89	0.04	0.3	10
1.83	0.09	0.4	10	1.41	0.07	0.4	10	1.66	0.11	0.4	10
2.03	0.05	0.8	10	1.89	0.05	0.8	10	2.02	0.06	0.8	10
1.63	0.27	0.1	20	1.18	0.11	0.1	20	1.86	0.52	0.1	20
1.91	0.15	0.2	20	1.41	0.10	0.2	20	1.76	0.04	0.2	20
1.90	0.09	0.3	20	1.50	0.08	0.3	20	1.81	0.02	0.3	20
1.94	0.07	0.4	20	1.75	0.06	0.4	20	1.99	0.08	0.4	20
1.91	0.04	0.8	20	1.81	0.04	0.8	20	1.88	0.04	0.8	20
1.83	0.25	0.1	40	1.48	0.13	0.1	40	2.09	0.06	0.1	40
2.27	0.12	0.2	40	1.82	0.10	0.2	40	2.16	0.02	0.2	40
2.14	0.09	0.3	40	1.89	0.08	0.3	40	2.00	0.01	0.3	40
1.86	0.07	0.4	40	1.73	0.06	0.4	40	1.77	0.01	0.4	40
1.98	0.04	0.8	40	1.95	0.04	0.8	40	1.87	0.01	0.8	40
2.04	0.24	0.1	80	1.54	0.16	0.1	80	1.99	0.04	0.1	80
2.24	0.12	0.2	80	2.00	0.11	0.2	80	2.12	0.01	0.2	80
1.91	0.08	0.3	80	1.77	0.07	0.3	80	1.77	0.01	0.3	80
1.91	0.06	0.4	80	1.81	0.06	0.4	80	1.76	0.01	0.4	80
1.93	0.03	0.8	80	1.91	0.03	0.8	80	1.83	0.00	0.8	80

Table 12. Logistic Regression Simulation β_2 results for models A, B and C and Ψ=1.0

Model A				Model B				Model C			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
2.41	0.40	0.1	5	0.80	0.09	0.1	5	-1.14	1.56	0.1	5
1.83	0.23	0.2	5	1.10	0.09	0.2	5	1.70	0.16	0.2	5
1.68	0.14	0.3	5	1.05	0.08	0.3	5	1.54	0.08	0.3	5
1.59	0.11	0.4	5	1.18	0.08	0.4	5	1.63	0.18	0.4	5
1.76	0.07	0.8	5	1.54	0.06	0.8	5	1.74	0.08	0.8	5
1.61	0.39	0.1	10	0.95	0.11	0.1	10	1.59	0.80	0.1	10
1.69	0.18	0.2	10	1.31	0.10	0.2	10	1.90	0.08	0.2	10
1.90	0.13	0.3	10	1.29	0.09	0.3	10	1.68	0.05	0.3	10
1.78	0.09	0.4	10	1.46	0.07	0.4	10	1.75	0.11	0.4	10
1.82	0.06	0.8	10	1.69	0.06	0.8	10	1.82	0.06	0.8	10
1.65	0.34	0.1	20	1.04	0.14	0.1	20	1.32	0.76	0.1	20
1.64	0.19	0.2	20	1.24	0.13	0.2	20	1.43	0.06	0.2	20
1.85	0.12	0.3	20	1.50	0.10	0.3	20	1.64	0.03	0.3	20
1.99	0.09	0.4	20	1.67	0.08	0.4	20	1.90	0.11	0.4	20
1.99	0.05	0.8	20	1.92	0.05	0.8	20	2.00	0.06	0.8	20
2.13	0.34	0.1	40	1.33	0.18	0.1	40	1.65	0.09	0.1	40
2.01	0.17	0.2	40	1.67	0.13	0.2	40	1.67	0.03	0.2	40
1.99	0.12	0.3	40	1.79	0.11	0.3	40	1.70	0.02	0.3	40
2.05	0.09	0.4	40	1.94	0.08	0.4	40	1.79	0.01	0.4	40
1.95	0.05	0.8	40	1.93	0.05	0.8	40	1.76	0.01	0.8	40

2.04	0.32	0.1	80	1.40	0.21	0.1	80	1.81	0.05	0.1	80
2.19	0.17	0.2	80	1.87	0.15	0.2	80	1.93	0.02	0.2	80
2.18	0.11	0.3	80	2.02	0.10	0.3	80	1.90	0.01	0.3	80
2.05	0.09	0.4	80	1.98	0.08	0.4	80	1.70	0.01	0.4	80
1.94	0.04	0.8	80	1.92	0.04	0.8	80	1.73	0.01	0.8	80

An increase in values of τ_{XX} lends higher reliabilities which lends to more accurate and precise estimates of the contextual effects. As soon as $\tau_{XX} = 0.2$ the values of the slopes and standard errors begin to show more stability, and approximate the true model parameter values much better than the model using the sample mean. The sample group mean values are inefficient estimates for the contextual effect (Fuller 1987), but they do not behave in the same manner as the EB estimates with the very low reliability as they do not display the large standard errors of the estimates. The reliability $\delta_j = \frac{\tau_{XX}}{\tau_{XX}+\frac{\sigma_{XX}{}^2}{Nj}}$ is affected by both the size of τ_{XX} and N_j, and as both are minimized, the standard errors and estimates of the slope parameter estimates grow and are less accurate. These patterns are also seen at all levels of Ψ. As τ_{XX} increases the estimates of the contextual effects are more accurate, but the EB method for contextual effects performs well as long as reliability is not extremely small. Tables 13-15 show the probit results for β_2 estimates in all three models for these simulations. Missing estimates are the result of the models not converging (this case was rare, and only with very large sample sizes).

Table 13. Probit Regression Simulation β_2 results for models A, B and C and Ψ=0.2

Model A				Model B				Model C			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
1.71	0.27	0.1	5	1.09	0.06	0.1	5	2.34	0.79	0.1	5
1.88	0.14	0.2	5	1.15	0.06	0.2	5	1.73	0.26	0.2	5
2.07	0.10	0.3	5	1.30	0.06	0.3	5	2.18	0.21	0.3	5
1.99	0.09	0.4	5	1.47	0.06	0.4	5	2.12	0.13	0.4	5
2.10	0.07	0.8	5	1.87	0.07	0.8	5	2.16	0.09	0.8	5
2.09	0.22	0.1	10	1.13	0.07	0.1	10	3.06	0.93	0.1	10
1.90	0.11	0.2	10	1.28	0.06	0.2	10	1.99	0.19	0.2	10
1.98	0.08	0.3	10	1.55	0.06	0.3	10	2.09	0.11	0.3	10
1.96	0.07	0.4	10	1.63	0.05	0.4	10	2.00	0.08	0.4	10

3.31	0.24	0.8	10	1.92	0.05	0.8	10	2.07	0.06	0.8	10
2.08	0.19	0.1	20	1.10	0.08	0.1	20	2.20	0.91	0.1	20
1.86	0.10	0.2	20	1.36	0.07	0.2	20	1.82	0.15	0.2	20
2.08	0.07	0.3	20	1.73	0.06	0.3	20	2.12	0.09	0.3	20
1.95	0.05	0.4	20	1.70	0.05	0.4	20	1.92	0.06	0.4	20
3.07	0.17	0.8	20	1.86	0.04	0.8	20	1.93	0.04	0.8	20
2.12	0.17	0.1	40	1.29	0.09	0.1	40	2.04	0.31	0.1	40
2.05	0.08	0.2	40	1.67	0.06	0.2	40	2.07	0.10	0.2	40
2.03	0.06	0.3	40	1.84	0.05	0.3	40	2.06	0.07	0.3	40
2.02	0.05	0.4	40	1.89	0.04	0.4	40	2.02	0.05	0.4	40
2.03	0.03	0.8	40	2.00	0.03	0.8	40	2.05	0.03	0.8	40
1.96	0.15	0.1	80	1.37	0.11	0.1	80	1.83	0.24	0.1	80
2.04	0.08	0.2	80	1.79	0.07	0.2	80	2.03	0.09	0.2	80
2.10	0.05	0.3	80	1.96	0.05	0.3	80	2.10	0.06	0.3	80
1.96	0.04	0.4	80	1.88	0.04	0.4	80	1.95	0.04	0.4	80
		0.8	80			0.8	80			0.8	80

Table 14. Probit Regression Simulation β_2 results for models A, B and C and Ψ=0.5

Model A				Model B				Model C			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
2.08	0.32	0.1	5	0.99	0.08	0.1	5	1.51	4.00	0.1	5
2.11	0.16	0.2	5	1.05	0.07	0.2	5	1.44	0.37	0.2	5
2.03	0.13	0.3	5	1.28	0.08	0.3	5	1.88	0.25	0.3	5
1.87	0.10	0.4	5	1.33	0.07	0.4	5	1.77	0.15	0.4	5
1.87	0.07	0.8	5	1.64	0.07	0.8	5	1.88	0.08	0.8	5
2.19	0.30	0.1	10	1.02	0.09	0.1	10	1.10	0.72	0.1	10
1.82	0.14	0.2	10	1.25	0.08	0.2	10	1.97	0.29	0.2	10
1.89	0.10	0.3	10	1.44	0.07	0.3	10	2.04	0.17	0.3	10
1.98	0.08	0.4	10	1.54	0.07	0.4	10	1.89	0.10	0.4	10
2.00	0.06	0.8	10	1.86	0.06	0.8	10	1.99	0.06	0.8	10
1.76	0.25	0.1	20	1.18	0.10	0.1	20	1.87	0.48	0.1	20
2.18	0.14	0.2	20	1.43	0.09	0.2	20	1.91	0.19	0.2	20
2.08	0.09	0.3	20	1.65	0.08	0.3	20	2.01	0.12	0.3	20
2.03	0.07	0.4	20	1.83	0.06	0.4	20	2.08	0.08	0.4	20
1.97	0.04	0.8	20	1.87	0.04	0.8	20	1.94	0.05	0.8	20
1.80	0.25	0.1	40	1.29	0.13	0.1	40	1.98	0.44	0.1	40
2.25	0.12	0.2	40	1.83	0.09	0.2	40	2.28	0.14	0.2	40
2.19	0.09	0.3	40	1.95	0.08	0.3	40	2.20	0.10	0.3	40
1.91	0.06	0.4	40	1.78	0.06	0.4	40	1.90	0.07	0.4	40
1.99	0.04	0.8	40	1.96	0.04	0.8	40	1.99	0.04	0.8	40
2.23	0.24	0.1	80	1.50	0.16	0.1	80	2.13	0.37	0.1	80
2.22	0.12	0.2	80	1.99	0.10	0.2	80	2.33	0.14	0.2	80
1.97	0.08	0.3	80	1.81	0.07	0.3	80	1.92	0.08	0.3	80
1.97	0.06	0.4	80	1.88	0.06	0.4	80	1.94	0.06	0.4	80
1.99	0.04	0.8	80	1.97	0.04	0.8	80	1.99	0.04	0.8	80

Table 15. Probit Regression Simulation β_2 results for models A, B and C and Ψ=1.0

Model A				Model B				Model C			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
2.45	0.38	0.1	5	0.96	0.09	0.1	5	0.27	1.44	0.1	5
2.06	0.22	0.2	5	1.23	0.09	0.2	5	1.90	0.50	0.2	5
2.01	0.15	0.3	5	1.29	0.08	0.3	5	1.97	0.26	0.3	5
1.92	0.12	0.4	5	1.47	0.08	0.4	5	2.04	0.18	0.4	5
2.01	0.09	0.8	5	1.77	0.08	0.8	5	2.01	0.10	0.8	5
1.68	0.39	0.1	10	1.01	0.11	0.1	10	1.10	0.79	0.1	10
1.82	0.18	0.2	10	1.36	0.10	0.2	10	2.11	0.33	0.2	10
2.19	0.13	0.3	10	1.50	0.09	0.3	10	2.10	0.21	0.3	10
2.17	0.11	0.4	10	1.77	0.08	0.4	10	2.18	0.13	0.4	10
2.03	0.07	0.8	10	1.89	0.07	0.8	10	2.04	0.07	0.8	10
1.64	0.34	0.1	20	1.03	0.14	0.1	20	1.23	0.76	0.1	20
1.77	0.19	0.2	20	1.27	0.13	0.2	20	1.68	0.31	0.2	20
1.92	0.12	0.3	20	1.54	0.09	0.3	20	1.87	0.15	0.3	20
2.09	0.09	0.4	20	1.76	0.08	0.4	20	2.01	0.11	0.4	20
2.08	0.06	0.8	20	2.04	0.06	0.8	20	2.12	0.06	0.8	20
1.96	0.33	0.1	40	1.24	0.18	0.1	40	1.89	0.66	0.1	40
2.03	0.17	0.2	40	1.65	0.13	0.2	40	2.06	0.22	0.2	40
2.01	0.12	0.3	40	1.81	0.11	0.3	40	2.05	0.14	0.3	40
2.05	0.09	0.4	40	1.97	0.08	0.4	40	2.13	0.10	0.4	40
2.05	0.05	0.8	40	2.04	0.05	0.8	40	2.08	0.05	0.8	40
2.10	0.33	0.1	80	1.52	0.22	0.1	80	2.22	0.51	0.1	80
2.38	0.18	0.2	80	2.02	0.15	0.2	80	2.36	0.20	0.2	80
2.22	0.11	0.3	80	2.07	0.11	0.3	80	2.22	0.12	0.3	80
2.05	0.09	0.4	80	1.99	0.08	0.4	80	2.06	0.09	0.4	80
2.03	0.05	0.8	80	2.00	0.05	0.8	80	2.02	0.05	0.8	80

These behaviors persist for each of the link functions produced here. More specifically, for the Logistic models we see as $N_j \geq 40$ the estimates of sample group means are reliable and thus the contextual effect estimate becomes stable at all levels of Ψ, but as $N_j<40$ the picture changes. With the very small group sizes ($N_j=5$) the estimates of the contextual effects across models A, B, and C are not even close, with large standard errors in model C in particular. As N_j grows and becomes large, the estimates are closer, but standard errors of those estimates can be large enough with small τ_{XX} we warn users to cautiously EB or sample mean method, as contextual effects methodology may not be applicable to a variable with no higher level variability (τ_{XX}). In these cases if the sample mean method is used, a biased estimate will still result.

For the probit model, the estimates of the contextual effects for models B and C are not as biased for the small τ, small N_j combination, but the standard errors are large enough that testing H_0: $\beta_2=0$ versus H_1: $\beta_2\neq0$ will not be significant ($\alpha=0.05$), and thus we should be cautious. As N_j grows above 40 the estimates are closer to the true value, and standard errors remain large for small τ_{xx}, but the larger N_j becomes, the smaller the standard errors, and thus the more unbiased the statistical inferences. Even as it is closer with larger N_j, $\tau_{xx}=0.1$ still has an effect of underestimating β_2 somewhat in models B and C. The amount of this underestimation varies in all models, and the final chapter examines the variation more extensively. These trends stay the same as Ψ grows for the probit analyses.

Poisson simulation studies produce very similar results as those of the probit simulations. The estimates of these log-linear models are all relatively stable as $N_j\geq40$, and even though the standard errors are a bit higher for small τ_{xx}, those of the β_2 estimates are not large. One difference seen in the Poisson models is that biased inferences could be made because of an underestimate of β_2 or by an unnaturally inflated estimate of β_2. These patterns are relatively the same for all three levels of Ψ that we study. For Logit, Probit and Poisson, we see that elevated τ_{xx} raises the reliability and thus enables the EB method to produce a much more accurate estimate of β_2 than the sample mean model, other than the few scenarios with issue. Tables 16-18 show the results of the Poisson regression simulations for the contextual effects.

Table 16. Poisson Regression Simulation β_2 results for models A, B and C and Ψ=0.2

Model A				Model B				Model C			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
2.11	0.17	0.1	5	1.02	0.04	0.1	5	1.20	0.50	0.1	5
1.85	0.08	0.2	5	1.19	0.03	0.2	5	1.89	0.15	0.2	5
2.02	0.06	0.3	5	1.33	0.04	0.3	5	2.18	0.12	0.3	5
2.03	0.04	0.4	5	1.44	0.03	0.4	5	2.00	0.08	0.4	5
2.01	0.03	0.8	5	1.75	0.03	0.8	5	2.01	0.04	0.8	5
2.05	0.15	0.1	10	1.12	0.05	0.1	10	2.63	0.66	0.1	10
1.95	0.08	0.2	10	1.31	0.04	0.2	10	1.99	0.14	0.2	10
1.94	0.05	0.3	10	1.41	0.04	0.3	10	1.83	0.07	0.3	10
1.99	0.04	0.4	10	1.63	0.03	0.4	10	2.03	0.05	0.4	10
1.99	0.02	0.8	10	1.85	0.02	0.8	10	1.99	0.03	0.8	10
1.98	0.14	0.1	20	1.12	0.06	0.1	20	2.34	0.69	0.1	20
1.95	0.08	0.2	20	1.38	0.05	0.2	20	1.87	0.12	0.2	20
2.04	0.05	0.3	20	1.68	0.04	0.3	20	2.04	0.06	0.3	20
1.95	0.04	0.4	20	1.71	0.04	0.4	20	1.93	0.05	0.4	20
1.98	0.02	0.8	20	1.90	0.02	0.8	20	1.97	0.02	0.8	20
2.19	0.14	0.1	40	1.31	0.08	0.1	40	2.04	0.27	0.1	40
2.10	0.07	0.2	40	1.67	0.06	0.2	40	2.07	0.09	0.2	40
2.04	0.05	0.3	40	1.83	0.04	0.3	40	2.05	0.06	0.3	40
2.03	0.04	0.4	40	1.90	0.04	0.4	40	2.03	0.04	0.4	40
2.01	0.02	0.8	40	1.98	0.02	0.8	40	2.02	0.02	0.8	40
1.96	0.14	0.1	80	1.38	0.10	0.1	80	1.86	0.22	0.1	80
1.98	0.07	0.2	80	1.75	0.06	0.2	80	1.99	0.08	0.2	80
2.08	0.05	0.3	80	1.95	0.05	0.3	80	2.09	0.05	0.3	80
1.96	0.04	0.4	80	1.88	0.04	0.4	80	1.95	0.04	0.4	80
2.00	0.02	0.8	80	1.98	0.02	0.8	80	2.00	0.02	0.8	80

Table 17. Poisson Regression Simulation β_2 results for models A, B and C and Ψ=0.5

Model A				Model B				Model C			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
2.23	0.24	0.1	5	1.04	0.05	0.1	5	3.18	2.89	0.1	5
2.18	0.12	0.2	5	1.17	0.05	0.2	5	2.01	0.28	0.2	5
1.95	0.08	0.3	5	1.27	0.05	0.3	5	1.96	0.17	0.3	5
1.88	0.06	0.4	5	1.38	0.04	0.4	5	1.86	0.09	0.4	5
1.92	0.03	0.8	5	1.72	0.03	0.8	5	1.96	0.05	0.8	5
1.93	0.23	0.1	10	1.04	0.07	0.1	10	1.34	0.57	0.1	10
1.83	0.11	0.2	10	1.22	0.06	0.2	10	1.83	0.23	0.2	10
1.89	0.08	0.3	10	1.44	0.06	0.3	10	2.04	0.14	0.3	10
1.97	0.06	0.4	10	1.54	0.05	0.4	10	1.87	0.08	0.4	10
2.03	0.03	0.8	10	1.88	0.03	0.8	10	2.01	0.04	0.8	10
1.71	0.23	0.1	20	1.21	0.09	0.1	20	2.02	0.43	0.1	20
2.06	0.12	0.2	20	1.42	0.08	0.2	20	1.89	0.17	0.2	20
2.06	0.08	0.3	20	1.63	0.06	0.3	20	2.00	0.10	0.3	20
2.04	0.06	0.4	20	1.82	0.05	0.4	20	2.07	0.06	0.4	20

1.98	0.03	0.8	20	1.87	0.03	0.8	20	1.94	0.03	0.8	20
1.99	0.23	0.1	40	1.41	0.12	0.1	40	2.35	0.41	0.1	40
2.23	0.11	0.2	40	1.83	0.08	0.2	40	2.29	0.13	0.2	40
2.15	0.08	0.3	40	1.91	0.07	0.3	40	2.15	0.09	0.3	40
1.95	0.06	0.4	40	1.83	0.06	0.4	40	1.97	0.07	0.4	40
2.03	0.03	0.8	40	2.00	0.03	0.8	40	2.04	0.03	0.8	40
2.19	0.23	0.1	80	1.51	0.15	0.1	80	2.15	0.35	0.1	80
2.25	0.11	0.2	80	2.01	0.10	0.2	80	2.36	0.14	0.2	80
1.96	0.07	0.3	80	1.81	0.07	0.3	80	1.92	0.08	0.3	80
1.96	0.05	0.4	80	1.87	0.05	0.4	80	1.93	0.06	0.4	80
1.99	0.03	0.8	80	1.97	0.03	0.8	80	1.99	0.03	0.8	80

Table 18. Poisson Regression Simulation β_2 results for models A, B and C and Ψ=1.0

Model A				Model B				Model C			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
2.48	0.32	0.1	5	1.02	0.07	0.1	5	1.21	1.19	0.1	5
2.02	0.17	0.2	5	1.20	0.07	0.2	5	2.08	0.38	0.2	5
1.88	0.11	0.3	5	1.21	0.06	0.3	5	1.70	0.20	0.3	5
1.82	0.08	0.4	5	1.39	0.06	0.4	5	1.91	0.14	0.4	5
2.01	0.05	0.8	5	1.80	0.04	0.8	5	2.05	0.06	0.8	5
1.76	0.33	0.1	10	1.03	0.10	0.1	10	1.22	0.69	0.1	10
1.98	0.16	0.2	10	1.39	0.09	0.2	10	2.27	0.28	0.2	10
2.06	0.11	0.3	10	1.40	0.08	0.3	10	1.89	0.17	0.3	10
2.11	0.08	0.4	10	1.73	0.06	0.4	10	2.13	0.10	0.4	10
1.99	0.04	0.8	10	1.91	0.04	0.8	10	2.05	0.05	0.8	10
1.72	0.32	0.1	20	1.09	0.13	0.1	20	1.49	0.70	0.1	20
1.82	0.17	0.2	20	1.28	0.12	0.2	20	1.69	0.29	0.2	20
1.97	0.11	0.3	20	1.59	0.09	0.3	20	1.92	0.13	0.3	20
2.03	0.08	0.4	20	1.72	0.07	0.4	20	1.96	0.10	0.4	20
2.05	0.04	0.8	20	1.99	0.04	0.8	20	2.07	0.04	0.8	20
1.96	0.32	0.1	40	1.23	0.17	0.1	40	1.85	0.64	0.1	40
1.99	0.16	0.2	40	1.64	0.13	0.2	40	2.06	0.21	0.2	40
2.02	0.11	0.3	40	1.81	0.10	0.3	40	2.04	0.13	0.3	40
2.07	0.08	0.4	40	1.98	0.08	0.4	40	2.14	0.09	0.4	40
1.99	0.04	0.8	40	1.95	0.04	0.8	40	1.99	0.04	0.8	40
2.08	0.32	0.1	80	1.49	0.22	0.1	80	2.12	0.50	0.1	80
2.35	0.17	0.2	80	2.00	0.15	0.2	80	2.33	0.19	0.2	80
2.20	0.11	0.3	80	2.06	0.10	0.3	80	2.21	0.12	0.3	80
2.04	0.08	0.4	80	1.97	0.08	0.4	80	2.05	0.09	0.4	80
2.00	0.04	0.8	80	1.98	0.04	0.8	80	2.00	0.04	0.8	80

In literature it is common to study contextual effects (as noted by citation in Shin and Raudenbush 2010, pg30) by using the sample mean model B, even in HGLM settings (Rabe

Hesketh 2009), but it is shown here that the estimate of the contextual effect will trend more towards the true effect as long as there is a reasonably high reliability in the group sample means of the predictor. Reliability ($\delta_j = \frac{\tau_{XX}}{\tau_{XX}+\frac{\sigma_{XX}{}^2}{Nj}}$) is a function of the individual variability in X but also the group to group variability, as well as the group sizes. Thus the adequacy of the EB mean $\delta_j * \bar{X}_j + (1 - \delta_j) * \widehat{r_x}$ depends on τ_{xx} as well as the group sizes. This relationship is more concretely established in the large scale simulations in the final chapter, but the behavior of estimation via each model may be described by studying the response surface of the model parameter estimates given the simulated parameters.

Another way to describe what is seen between the changes across the model parameter estimates with the simulated parameters is in terms of the size of the predictor (X) and the size of the outcome variable. What this means is that more variability in either or both of these allows the slope to be better estimated, as a very small leverage of either makes the relationship harder to estimate. The larger values of τ_{XX} tend to allow more leverage within values of X at the higher level, while a small τ_{XX} gives a much smaller window of variation for the values of X, or rather, X cannot take on as many values, making the relationship more difficult to estimate. There is a similar phenomenon with varying values of Ψ. That is, if the span (or variation) in the outcome values is small, the values of the outcome are not as widespread which leads to a difficulty in finding the relationships between the outcome and predictor. This is a very similar issue to that of principal component analysis, where you attempt to iteratively find the largest axis of variation. When the variation of different variables in this setting is similar, then it is not clear as to which direction produces the largest variation. In the simulation here we study the stability of estimation of contextual effects by the models A, B, and C as the sizes of the variance components (Ψ and τ_{XX}) change.

Two other generalized linear models that we wish to study are those of ordinal data and categorical data. The ordinal data is a situation where the outcome has categories with an inherent order. We simulated the model via a latent variable that determines the underlying ordering mechanism for this model, and the model will be simulated within the realm of the proportional odds assumption. Nominal logistic regression is a common analysis technique when one wants to fit a set of linear models to non-ordered categorical data. A reference category is chosen, and then a regression model is fitted to compare the odds of being in one of the categories versus the referent. Below we show how this is done to a more intricate degree.

2.3 Ordinal Probit Simulation

The next generalized linear model setting studied is that of the cumulative probit model. This setting is that of which the outcome is categorical, but has an inherent (underlying) order to it. These variables are seen as having multiple groups, but there being an order to the groups. One way to look at this is that there is a latent variable ordering the groups, and that thresholds exist as values of the latent variable dividing population units into the ordered groups. As the method of fitting a linear model to the ordinal data is explained further, the three models that we will compare have the following appearance:

Model A: $Y_{ij} = \Phi(\mathrm{l_g} + \beta_1 * \left(X_{ij} - \mu_{X_j}\right) + \beta_2 * \mu_{X_j} + u_j)$

Model B: $Y_{ij} = \Phi(\mathrm{l_g}^{\mathrm{B}} + \beta_1{}^{B} * \left(X_{ij} - \bar{X}_j\right) + \beta_2{}^{B} * \bar{X}_j + u_j)$

Model C: $Y_{ij} = \Phi(\mathrm{l_g}^{\mathrm{C}} + \beta_1{}^{\mathrm{C}} * \left(\mathrm{X_{ij}} - \mu_{\mathrm{X_j}}^{\mathrm{EB}}\right) + \beta_2{}^{\mathrm{C}} * \mu_{\mathrm{X_j}}^{\mathrm{EB}} + u_j)$.

The terms β_1 and β_2 are interpreted as before – which is allowed by the proportional odds assumption saying that the rate of change based on X is the same for all outcome groups. However, there are multiple intercepts $\mathrm{l_g}$ (g=1,…,G-1) where G is the number of categories in the ordered categorical outcome variable. The random term u_{1j} is interpreted as before which is the level-2 variance component with expectation zero and variance Ψ. The fact that this model has one intercept per group (the intercept being zero for the reference group), and only the one slope for each of the predictors shows the proportional odds assumption. The way we simulate this model is to calculate the following latent L_{ij} from the simulated predictor X:

$$L_{ij} = \eta_{ij} + e_{ij} = 1 * \left(X_{ij} - \mu_{X_j}\right) + 2 * \mu_{X_j} + u_{1j} + e_{ij}, where\ e_{ij} \sim N(0,1)$$

For this illustration/study we assume 3 groups that are divided using L_{ij} as follows:

$$Y_{ij} = 0 \leftrightarrow L_{ij} < l_1$$

$$Y_{ij} = 1 \leftrightarrow l_1 \leq L_{ij} < l_2$$

$$Y_{ij} = 2 \leftrightarrow l_2 \leq L_{ij}$$

This shows the latent variable approach to regarding the ordinal variable in a cumulative probability setting. This is also outlined by Gueorguieval and Sanacora (2006), and we mostly follow their lead on the formulation for the ordinal model for this simulation. This says that a normal random variable L_{ij} is latent to the group ordering, shown by the cutoffs l_1 and l_2. For our purposes, we will set l_1=-1 and l_2=2. In estimating the three models shown above we estimate the true parameters (arbitrarily chosen): $\beta_1 = 1$, $\beta_2 = 2$, $l_1 = -1$, $l_2 = 2$ and Ψ. The likelihood that is maximized with respect to these parameters using Gaussian Quadrature is defined by:

$$\prod_j \prod_i {p_0}^{I(Y_{ij}=0)} * (p_1 - p_0)^{I(Y_{ij}=1)} * (1 - p_1)^{I(Y_{ij}=2)}$$

Where $p_g = \Phi\left(l_{g+1} - \eta_{ij}\right)$ where η_{ij} is the linear component given random effects u_j in the model A, B, or C. The functions ($I(Y_{ij}$=g)) are indicator functions of category g, showing the contribution to the likelihood from each outcome group.

Table 19. Ordinal Probit Regression Simulation β_2 results for models A, B and C and Ψ=0.2

Model A				Model B				Model C			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
2.03	0.24	0.1	5	1.08	0.06	0.1	5	1.87	0.72	0.1	5
1.91	0.12	0.2	5	1.24	0.05	0.2	5	2.19	0.23	0.2	5
1.99	0.09	0.3	5	1.30	0.05	0.3	5	2.15	0.18	0.3	5
2.00	0.07	0.4	5	1.41	0.05	0.4	5	1.97	0.11	0.4	5
2.00	0.05	0.8	5	1.44	0.03	0.8	5	1.69	0.05	0.8	5
2.03	0.20	0.1	10	1.24	0.06	0.1	10	3.89	0.86	0.1	10
1.83	0.10	0.2	10	1.29	0.05	0.2	10	1.93	0.17	0.2	10
1.87	0.07	0.3	10	1.38	0.05	0.3	10	1.75	0.10	0.3	10
1.72	0.05	0.4	10	1.46	0.04	0.4	10	1.83	0.07	0.4	10
1.60	0.03	0.8	10	1.84	0.04	0.8	10	1.99	0.04	0.8	10
1.92	0.17	0.1	20	1.10	0.07	0.1	20	2.00	0.82	0.1	20
1.87	0.09	0.2	20	1.36	0.06	0.2	20	1.82	0.14	0.2	20
2.00	0.06	0.3	20	1.65	0.05	0.3	20	2.01	0.07	0.3	20
1.94	0.05	0.4	20	1.52	0.04	0.4	20	1.73	0.06	0.4	20
2.02	0.03	0.8	20	1.41	0.02	0.8	20	1.46	0.03	0.8	20
2.19	0.16	0.1	40	1.31	0.09	0.1	40	2.02	0.30	0.1	40
2.12	0.08	0.2	40	1.67	0.06	0.2	40	2.07	0.10	0.2	40
1.94	0.06	0.3	40	1.75	0.05	0.3	40	1.96	0.06	0.3	40
2.06	0.04	0.4	40	1.93	0.04	0.4	40	2.07	0.05	0.4	40
1.41	0.02	0.8	40	1.39	0.02	0.8	40	1.42	0.02	0.8	40
1.96	0.15	0.1	80	1.41	0.10	0.1	80	1.92	0.23	0.1	80
2.03	0.07	0.2	80	1.79	0.08	0.2	80	2.04	0.09	0.2	80
2.08	0.05	0.3	80	1.95	0.05	0.3	80	2.08	0.06	0.3	80
1.97	0.04	0.4	80	1.89	0.04	0.4	80	1.96	0.04	0.4	80
1.98	0.02	0.8	80	1.96	0.02	0.8	80	1.98	0.02	0.8	80

Table 20. Ordinal Probit Regression Simulation β_2 results for models A, B and C and Ψ=0.5

Model A				Model B				Model C			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
1.90	0.28	0.1	5	0.99	0.07	0.1	5	2.42	3.44	0.1	5
1.97	0.15	0.2	5	1.07	0.06	0.2	5	1.24	0.33	0.2	5
1.88	0.10	0.3	5	1.27	0.06	0.3	5	1.79	0.20	0.3	5
1.89	0.08	0.4	5	1.37	0.06	0.4	5	1.82	0.12	0.4	5
1.99	0.06	0.8	5	1.74	0.05	0.8	5	1.97	0.07	0.8	5
1.86	0.26	0.1	10	1.00	0.08	0.1	10	1.14	0.64	0.1	10
1.88	0.13	0.2	10	1.22	0.07	0.2	10	1.79	0.27	0.2	10
1.98	0.09	0.3	10	1.50	0.07	0.3	10	2.18	0.15	0.3	10
2.00	0.07	0.4	10	1.60	0.06	0.4	10	1.96	0.09	0.4	10
1.45	0.03	0.8	10	1.40	0.03	0.8	10	1.50	0.04	0.8	10
1.56	0.25	0.1	20	1.19	0.10	0.1	20	1.94	0.46	0.1	20
2.10	0.13	0.2	20	1.42	0.08	0.2	20	1.92	0.18	0.2	20
1.81	0.08	0.3	20	1.56	0.07	0.3	20	1.75	0.10	0.3	20
1.74	0.06	0.4	20	1.66	0.05	0.4	20	1.88	0.07	0.4	20

1.98	0.04	0.8	20	1.47	0.03	0.8	20	1.53	0.03	0.8	20
1.90	0.23	0.1	40	1.41	0.13	0.1	40	2.39	0.42	0.1	40
1.80	0.11	0.2	40	1.55	0.08	0.2	40	1.90	0.13	0.2	40
1.82	0.08	0.3	40	1.62	0.07	0.3	40	1.81	0.09	0.3	40
1.63	0.06	0.4	40	1.52	0.05	0.4	40	1.62	0.06	0.4	40
1.44	0.03	0.8	40	1.37	0.03	0.8	40	1.39	0.03	0.8	40
1.81	0.22	0.1	80	1.27	0.15	0.1	80	1.86	0.00	0.1	80
1.85	0.10	0.2	80	1.65	0.09	0.2	80	1.93	0.12	0.2	80
1.64	0.06	0.3	80	1.52	0.06	0.3	80	1.61	0.07	0.3	80
1.63	0.05	0.4	80	1.53	0.05	0.4	80	1.58	0.05	0.4	80
1.99	0.03	0.8	80	1.98	0.03	0.8	80	2.01	0.03	0.8	80

Table 21. Ordinal Probit Regression Simulation β_2 results for models A, B and C and Ψ=1.0

Model A				Model B				Model C			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
2.69	0.37	0.1	5	1.08	0.09	0.1	5	2.01	1.37	0.1	5
2.00	0.20	0.2	5	1.20	0.08	0.2	5	2.19	0.45	0.2	5
1.96	0.13	0.3	5	1.22	0.08	0.3	5	1.78	0.25	0.3	5
1.74	0.09	0.4	5	1.37	0.07	0.4	5	1.77	0.15	0.4	5
1.42	0.05	0.8	5	1.28	0.04	0.8	5	1.53	0.06	0.8	5
1.76	0.37	0.1	10	1.05	0.10	0.1	10	1.31	0.75	0.1	10
1.95	0.17	0.2	10	1.42	0.10	0.2	10	2.07	0.31	0.2	10
1.89	0.12	0.3	10	1.36	0.09	0.3	10	1.71	0.19	0.3	10
1.51	0.07	0.4	10	1.25	0.06	0.4	10	1.49	0.09	0.4	10
1.46	0.04	0.8	10	1.34	0.04	0.8	10	1.45	0.04	0.8	10
1.32	0.30	0.1	20	0.98	0.13	0.1	20	1.23	0.67	0.1	20
1.32	0.14	0.2	20	0.93	0.10	0.2	20	1.25	0.24	0.2	20
1.54	0.10	0.3	20	1.22	0.08	0.3	20	1.44	0.12	0.3	20
1.45	0.07	0.4	20	1.23	0.06	0.4	20	1.40	0.09	0.4	20
1.45	0.04	0.8	20	1.49	0.04	0.8	20	1.52	0.04	0.8	20
1.58	0.30	0.1	40	0.92	0.16	0.1	40	1.31	0.61	0.1	40
1.41	0.12	0.2	40	1.16	0.10	0.2	40	1.44	0.16	0.2	40
1.44	0.11	0.3	40	1.29	0.10	0.3	40	1.44	0.12	0.3	40
1.62	0.07	0.4	40	1.53	0.07	0.4	40	1.63	0.09	0.4	40
1.45	0.03	0.8	40	1.49	0.03	0.8	40	1.52	0.03	0.8	40
1.42	0.25	0.1	80	1.01	0.17	0.1	80	1.39	0.39	0.1	80
1.65	0.12	0.2	80	1.40	0.11	0.2	80	1.62	0.14	0.2	80
1.69	0.11	0.3	80	1.58	0.10	0.3	80	1.68	0.12	0.3	80
1.60	0.08	0.4	80	1.56	0.08	0.4	80	1.60	0.09	0.4	80
2.01	0.04	0.8	80	1.98	0.04	0.8	80	2.00	0.04	0.8	80

In the results shown in tables 19-21 (β_2 only, the remainder in the appendix) we see the estimates of β_1 and β_2 and Ψ as before, but now look at the two intercept terms, and see how close the estimates are to the true values in the simulation. The results largely show the same phenomenon we saw before with the low reliability relating to the inability to estimate the contextual effect using the Empirical Bayes approach. With sizable reliability for the group means of X_{ij}, however, the Empirical Bayes estimation outperforms the sample mean for estimating the true contextual effect. As shown in the literature for normal linear models, the within effect is the same for the sample mean model and the EB model, while the effect for the contextual effect is better estimated by the EB method. The intercept terms (two for three ordered groups) do not remain the same (as expected) but are closely estimated across models A, B and C. The level two variability estimate of Ψ is not always accurately estimated, however. In some cases, it is seen that the estimates of this variance component are not as close to the true values as expected. This is mostly seen only when the true value of Ψ is large in all models A, B and C, in this case 1 – which is the same as the level one variability induced in the probit analysis.

In terms of Ψ and τ_{xx} there are unique findings to the ordinal analysis. We see as we have before that when τ_{xx} is small (τ_{xx}=0.1) the contextual effect is not accurately estimating the true value and also have large standard errors making them more susceptible to biased inferences. We see that the true model is not correctly estimating the value of β_2 or β_1 due to the small leverage of X_{ij}. Given that, the parameter estimates via Model B are not close to those of the true model, and are underestimates, as we have seen previously. The EB Model C estimates the true model much closer, and even come closer to the true values in some situations than the true model itself. This behavior happens by chance because it can be seen that all three models underestimate the true values simulated, and caution should be used in this case.

2.4 Nominal Model Simulation

The next case is a nominal outcome with G categories. This situation occurs when the outcome of interest is a categorical (or grouping) variable that does not have an order. The link function for this scenario is the generalized logit, where there is a reference group, and there are G-1 logistic regression models where each of the G groups is compared to the reference group. A linear model is fit for each comparison. Here it will be assumed that there are G=3 nominal outcome categories. For our simulation we use the X data as we have all along, and simulate the probabilities that each observation is in each of the three groups. This is done using the constraint:

$$p_2 = 1 - p_0 - p_1$$

Again, using η_{0ij} and η_{1ij} as the linear predictors given level-2 random effects we have the following two models of interest in this situation:

Model A – True/Known Model

$$\log\left(\frac{p_{0ij}}{p_{2ij}}\right) = \eta_{0ij} = \beta_{01} + \beta_{11} * \left(\mathrm{X_{ij}} - \mu_{X_j}\right) + \beta_{21} * \mu_{X_j} + u_{1j}$$

$$\log\left(\frac{p_{1ij}}{p_{2ij}}\right) = \eta_{1ij} = \beta_{02} + \beta_{12} * \left(\mathrm{X_{ij}} - \mu_{X_j}\right) + \beta_{22} * \mu_{X_j} + u_{2j}$$

For the exercise, we set two different sets of parameters for the model. The simulation parameters are set (arbitrarily, made to be noticeably different to the eye) to: $\beta_{01} = 1, \beta_{11} = 1, \beta_{21} = 2, \beta_{02} = 2, \beta_{12} = 3, \beta_{22} = 5$. So to solve for the random probabilities, we solve the following system of equations for $p_{0ij}$and p_{1ij},:

$$\frac{p_{0ij}}{p_{2ij}} = \frac{p_{0ij}}{1 - p_{0ij} - p_{1ij}} = \exp\left(\eta_{0ij}\right)$$

$$\frac{p_{1ij}}{p_{2ij}} = \frac{p_{1ij}}{1 - p_{0ij} - p_{1ij}} = \exp(\eta_{1ij})$$

the algebraic result of which yields the following fact:

$$p_{0ij} = \frac{\exp(\eta_{0ij})}{1 + \exp(\eta_{0ij}) + \exp(\eta_{1ij})} \; and \; p_{1ij} = \frac{\exp(\eta_{1ij})}{1 + \exp(\eta_{0ij}) + \exp(\eta_{1ij})}.$$

The outcome variable, Y_{ij}, is then simulated by drawing a sample from a random multinomial variable with probabilities (p_{0ij}, p_{1ij}, and $1-p_{0ij}-p_{1ij}$) for each individual i in group j. In the simulation we will vary τ_{XX} and Ψ and N_j as before, but in this case we have a Ψ_1 and a Ψ_2 for the two generalized logits. We will look at situations of combinations of two values for the variance components for Ψ_1 and Ψ_2. The situations with small variance of Ψ do not show reasonable estimates, so the situation of two large variance components Ψ_1, Ψ_2=1 and the situation with one larger than the other Ψ_1=0.5 and a Ψ_2=1 will be simulated. The group sizes (N_j) will not be shown higher than 20, as it is seen that group sizes greater than or equal to 20 begin to converge in estimates to the true values across different models. The models that will be fit are model A shown immediately above using the known generated group means, and models B and C as follows:

Model B – Sample Mean Model:

$$\log\left(\frac{p_{0ij}}{p_{2ij}}\right) = \beta_{01}{}^{B} + \beta_{11}{}^{B} * \left(X_{ij} - \bar{X}_j\right) + \beta_{21}{}^{B} * \bar{X}_j + u_{1j}$$

$$\log\left(\frac{p_{1ij}}{p_{2ij}}\right) = \beta_{02}{}^{B} + \beta_{12}{}^{B} * \left(X_{ij} - \bar{X}_j\right) + \beta_{22}{}^{B} * \bar{X}_j + u_{2j}$$

Model C – Empirical Bayes Model:

$$\log\left(\frac{p_{0ij}}{p_{2ij}}\right) = \beta_{01}{}^{C} + \beta_{11}{}^{C} * \left(X_{ij} - \mu_{X_j}^{EB}\right) + \beta_{21}{}^{C} * \mu_{X_j} + u_{1j}$$

$$\log\left(\frac{p_{1ij}}{p_{2ij}}\right) = \beta_{02}{}^{C} + \beta_{12}{}^{\mathrm{C}} * \left(\mathrm{X_{ij}} - \mu_{\mathrm{X_j}}^{\mathrm{EB}}\right) + \beta_{22}{}^{\mathrm{C}} * \mu_{X_j} + u_{2j}$$

Table 22. Nominal Regression β_{21} and β_{22} results for models A, B and C and Ψ=(1.0, 1.0)

Model A (β_{21})				Model B (β_{21})				Model C (β_{21})			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
2.10	0.61	0.1	5	0.86	0.14	0.1	5	-1.35	1.83	0.1	5
1.89	0.17	0.4	5	1.40	0.12	0.4	5	1.80	0.26	0.4	5
1.93	0.12	0.8	5	1.67	0.10	0.8	5	1.94	0.13	0.8	5
2.68	0.48	0.1	10	1.23	0.15	0.1	10	4.53	2.04	0.1	10
2.04	0.14	0.4	10	1.55	0.11	0.4	10	1.83	0.18	0.4	10
2.06	0.09	0.8	10	1.80	0.09	0.8	10	1.93	0.10	0.8	10
1.29	0.39	0.1	20	1.11	0.17	0.1	20	2.56	1.88	0.1	20
2.03	0.12	0.4	20	1.69	0.11	0.4	20	1.91	0.14	0.4	20
2.00	0.07	0.8	20	1.89	0.07	0.8	20	1.96	0.07	0.8	20
Model A (β_{22})				Model B (β_{22})				Model C (β_{22})			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
4.53	0.63	0.1	5	3.03	0.16	0.1	5	1.92	1.87	5	0.1
4.91	0.22	0.4	5	3.90	0.16	0.4	5	4.74	0.31	5	0.4
5.33	0.19	0.8	5	4.82	0.18	0.8	5	5.37	0.21	5	0.8
4.91	0.50	0.1	10	3.09	0.16	0.1	10	4.95	2.13	10	0.1
4.97	0.16	0.4	10	4.12	0.13	0.4	10	4.79	0.19	10	0.4
5.18	0.13	0.8	10	4.80	0.12	0.8	10	5.07	0.13	10	0.8
5.14	0.41	0.1	20	3.38	0.18	0.1	20	7.48	1.98	20	0.1
5.00	0.13	0.4	20	4.43	0.11	0.4	20	4.88	0.14	20	0.4
4.92	0.09	0.8	20	4.76	0.09	0.8	20	4.89	0.09	20	0.8

Table 23. Nominal Regression β_{21} and β_{22} results for models A, B and C and Ψ=(0.5, 1.0)

Model A (β_{21})				Model B (β_{21})				Model C (β_{21})			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
1.27	0.47	0.1	5	0.87	0.12	0.1	5	0.01	1.53	0.1	5
2.06	0.15	0.4	5	1.46	0.11	0.4	5	2.20	0.23	0.4	5
1.87	0.10	0.8	5	1.62	0.09	0.8	5	1.87	0.11	0.8	5
2.30	0.37	0.1	10	1.26	0.12	0.1	10	4.90	1.63	0.1	10
2.07	0.12	0.4	10	1.66	0.09	0.4	10	2.05	0.14	0.4	10
2.04	0.08	0.8	10	1.86	0.07	0.8	10	2.01	0.08	0.8	10
2.01	0.29	0.1	20	1.20	0.13	0.1	20	2.72	1.41	0.1	20
1.88	0.09	0.4	20	1.62	0.08	0.4	20	1.83	0.10	0.4	20
1.96	0.06	0.8	20	1.88	0.06	0.8	20	1.95	0.06	0.8	20
Model A (β_{22})				Model B (β_{22})				Model C (β_{22})			
Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj	Est	SE	τ_{xx}	Nj
4.56	0.64	0.1	5	3.03	0.16	0.1	5	2.51	1.87	0.1	5

4.77	0.20	0.4	5	3.88	0.15	0.4	5	5.13	0.29	0.4	5
5.06	0.17	0.8	5	4.46	0.15	0.8	5	5.00	0.18	0.8	5
4.50	0.49	0.1	10	3.30	0.16	0.1	10	7.13	2.09	0.1	10
5.06	0.16	0.4	10	4.26	0.13	0.4	10	4.98	0.19	0.4	10
4.86	0.12	0.8	10	4.58	0.11	0.8	10	4.86	0.12	0.8	10
5.07	0.40	0.1	20	3.47	0.18	0.1	20	8.06	1.92	0.1	20
5.09	0.13	0.4	20	4.55	0.12	0.4	20	5.02	0.15	0.4	20
5.02	0.09	0.8	20	4.84	0.09	0.8	20	4.98	0.09	0.8	20

The resulting studies (results for shown in Tables 22 and 23) of the nominal models show similar findings to that of what has been seen to this point. It is supported again here that use of the sample group means in Model B produce biased estimates for the contextual effects. The Empirical Bayes estimates perform better with estimates closer to the values of the true values. However, small values of the variance in X again leads to difficulty in estimating the between effect (also contextual effect) using the Empirical Bayes approach, which is indicated with large standard errors and often abnormally larger estimates than the true contextual effects. However, with larger values of τ_{XX} the estimates are stable in both generalized logit models for the slope parameters in the EB method. The sample mean model B produces an underestimated contextual effect throughout, and is not as affected by the variability of X, as reliability does not affect estimates as much as it does in the EB method. The estimates of the variance component are very close to the true values when both u_1 and u_2 are large, but when one is small and the other is large, the larger one leads to very close estimates, whereas the small component seems to be very much underestimated.

2.5 Description of Appendices

The results of this chapter are contained in the appendix. Tabulated results for simulations of Logit, Probit, Poisson, Ordinal and Nominal models are contained. SAS programs for the execution of the simulations are also contained in Appendices. All SAS programs are written and executed in version 9.3.

Chapter 3

Large-Scale Simulations for the study of Contextual Effects in Logistic and Probit Regression Models

3.1 Explanation

In this final section, we aim to solidify findings of the simulations in the previous section by repeating certain iterations of the simulations many (m) times. That is, we will take a combination of N_j, τ_{xx}, Ψ, (as introduced in the preceding chapter) and repeat the process detailed previously and study the results when this process is repeated m times. Using a specific set of these parameters, we generate the predictor variable, X, then an outcome variable distributed as a pre-specified non-Normal, known distribution from the exponential family, and then fit a two-level HGLM (HGLM2) to the data with the three models. The HGLM2 will use the specified non-linear link function from the exponential family distribution. The three models A, B, and C from Chapter 2 will again be fit to the true (known, simulated) means, the sample group means of X ($\overline{X_J}$), and the EB means (μ_j^{EB}), as outlined previously. Repeating the simulations will show the expected value of the model parameter estimates under a set of values by the simulation. That is, using 95% confidence intervals for the contextual effect in each model, the aim is to see if the true model parameter is contained in the intervals, and to produce the coverage of the true value by the intervals.

The sets of parameters that will be used are combinations of the following

- τ_{xx} = 0.1, 0.4, 0.8
- Ψ = 0.2, 0.5, 1.0
- N_j = 5, 10, 20
- J=1000
- m=1000

The above parameters are used to estimate a simulated model 27 times, one for each of the combinations of (τ_{xx}, Ψ, and N_j). The simulations will be executed with both Logit link functions and Probit Link functions for binary data. These impose different assumptions on the binary data, but are both commonly used for modeling probabilities, proportions, and other binary data. To remind the reader of the three models we fit, they are listed below, with Model A being the true model, Model B being the sample mean model and Model C being the EB mean model. Expressed below is the logistic regression form of the models:

$$\text{Model A: } Logit\left(Y_{ij}\right) = \beta_0 + \beta_1 * \left(X_{ij} - \mu_{X_j}\right) + \beta_2 * \mu_{X_j} + u_j$$

$$\text{Model B: } Logit\left(Y_{ij}\right) = \beta_0 + \beta_1 * \left(X_{ij} - \bar{X}_j\right) + \beta_2 * \bar{X}_j + u_j$$

$$\text{Model C: } Logit\left(Y_{ij}\right) = \beta_0 + \beta_1 * \left(X_{ij} - \mu_{X_j}^{EB}\right) + \beta_2 * \mu_{X_j}^{EB} + u_j.$$

The goal of these simulations will be to study how closely the values of the estimates of β_0, β_1, β_2, and the variance component of u_j are to the values over many runs of simulated possibilities of X and the outcome Y for all three models. As was done in the less extensive runs of the simulations, we use Gaussian Quadrature to maximize the likelihood of these distributions and obtain the parameters in each model. The advantages of the Gaussian Quadrature estimation are the exactness of the numerical integration, as outlined by Rabe-Hesketh in GLLAMM.

When an iteration of the simulation is completed, we record each parameter estimate and the variance component estimate, as well as their standard errors. When 1000 model fits have been completed the average value of the estimate is recorded, as well as the average standard error for the estimate, the average upper and lower 95% normal confidence limits for the parameter (using the estimate and standard error along with a critical value from the N(0,1) distribution), as well as the number of times out of 1000 model fits (in percent) that the confidence interval contains the true value. The percent of times the true value of the parameter is contained in the confidence intervals will be called the coverage probability or percentage.

Note: The estimates for the variance component, say σ^2, over the m estimations of a simulated model are likely to have a skewed distribution. Because of this, the confidence interval estimates are calculated on $\log(\sigma^2)$ assumed to be normally distributed using the delta-method for calculation of the standard error. The confidence limits for the log of the variance component estimate are then exponentiated (inverse of logarithm) to obtain the proper confidence interval estimates.

For reference (as transcribed in more detail in the previous chapter), the following will be repeated 1000 times for each of the 27 combinations of parameter values. These are the steps for the logistic regression simulations.

1) Simulate $X_{ij} = \mu_{X_j} + e_{X_j}$, where $e_{X_j} \sim N(0, \sigma_{xx}^2 \equiv 1)$ and $\mu_{X_j} = r_X + u_{X_j}$ for $\mu_{X_j} \sim N(r_X, \tau_{XX})$

2) Simulate p_{ij} such that

$$Logit\left(p_{ij}\right) = \log\left(\frac{p_{ij}}{1-p_{ij}}\right) = \beta_0 + \beta_1 * \left(X_{ij} - \mu_{X_j}\right) + \beta_2 * \mu_{X_j} + u_j,$$

Where parameters for the simulation are set such that: $\beta_0, \beta_1 = 1$ and $\beta_2 = 2$, and $u_j \sim N(0, \Psi)$.

3) Simulate the binary outcome, $Y_{ij} = Binomial(p_{ij})$ for level-1 variability.

4) Fit Models A, B, and C to the outcome.

Again to be concise, the following steps will be repeated 1000 times for each of the 27 combinations of parameter values for the Probit models.

For the binary data with a probit link, steps 2, 3, and 4 are as follows:

1) Simulate $X_{ij} = \mu_{X_j} + e_{X_j}$, where $e_{X_j} \sim N(0, \sigma^2 \equiv 1)$ and $\mu_{X_j} = r_X + u_{X_j}$ for $\mu_{X_j} \sim N(r_X, \tau_{XX})$.

2) Simulate p_{ij} such that $\Phi^{-1}(p_{ij}) = \beta_0 + \beta_1 * \left(X_{ij} - \mu_{X_j}\right) + \beta_2 * \mu_{X_j} + u_{1j}$, or,

Where parameters for the simulation are set such that: $\beta_0, \beta_1 = 1$ and $\beta_2 = 2$, $u_{1j} \sim N(0, \Psi)$, and $e_{ij} \sim N(0,1)$.

3) Simulate the binary outcome, $Y_{ij} = Binomial(p_{ij})$.

4) Fit Models A, B, and C to the outcome.

The following section describes what was seen in these simulations, mostly concentrating on the coverage percentages as indication of estimation performance. Summary tables in the appendix will show resulting mean values, where accuracy can also be judged and used for reference in similar analyses.

3.2 Results

For the volume of results for this type of simulation, we will present the results in a table as well as in text. The results vary obviously by the values of the three parameters (Ψ, τ_{xx}, N_j), but it is informative to look at the results by combinations of variance ratios. Namely the effect of τ_{xx} is best seen as two ratios; τ_{xx}/σ and τ_{xx}/Ψ. This way the ratio is studied as the size of τ_{xx}, the upper level variability in X, in regards to the lower level variability of X (σ). It is also seen as the size of τ_{xx} in reference to the size of the remaining higher level variability in the outcome (Ψ). These also vary based on the group sample sizes (N_j), so the following table (Table 10 – Similar form to Table 4 of Olsen and Schafer 2001) has been constructed to show the coverages of β_2, the level-two slope, for every one of the 27 simulations performed for both the Logit models and the Probit models. Recall β_2 is the between effect (as defined previously), but as the level one slope β_1 estimates do not change across models, the changes in β_2 estimates are directly related to the changes in estimates of β_c, the contextual effect.

Tables 24 and 25 show the average lower and upper confidence limits on the β_2 estimates in the large scale simulations. These tables show the suspected patterns, where with low τ_{xx} we see wider confidence intervals (thus higher standard errors) which can lead to inaccurate results and biased inference on the contextual effect of the variable. Shown in these tables also is that the sample mean model B does not have the large standard error problems in smaller τ_{xx} but is a biased estimate of the true effect. As τ_{xx} increases the EB model C has unbiased estimates of the true effect, and thus would allow the researcher proper inference and study of this effect. The sample mean model B has biased estimates of the true effect throughout, though the bias decreases as expected with higher N_j, τ_{xx}, and Ψ.

Table 24. Average Confidence Intervals on β_2 estimates in the large scale simulations for Logistic Regression Contextual Effects Models

Simulation Parameters			Model A		Model B		Model C	
τ_{xx}	Ψ	Nj	Avg LL	Avg UL	Avg LL	Avg UL	Avg LL	Avg UL
0.1	0.2	5	1.26	2.72	0.91	1.09	-5.58	17.02
0.1	0.2	10	1.46	2.56	0.92	1.26	-0.84	5.87
0.1	0.2	20	1.56	2.43	0.98	1.35	0.88	3.22
0.1	0.5	5	1.18	2.81	0.86	1.23	-6.71	17.13
0.1	0.5	10	1.35	2.66	0.9	1.3	4.01	48.5
0.1	0.5	20	1.44	2.56	0.94	1.4	0.59	3.6
0.1	1	5	1.06	2.95	0.84	1.26	-6.86	14.46
0.1	1	10	1.19	2.79	0.85	1.34	-1.93	7.08
0.1	1	20	1.27	2.71	0.88	1.46	0.18	4.06
0.4	0.2	5	1.79	2.21	1.3	1.59	1.69	2.34
0.4	0.2	10	1.84	2.15	1.49	1.74	1.8	2.2
0.4	0.2	20	1.88	2.12	1.65	1.87	1.86	2.14
0.4	0.5	5	1.77	2.23	1.29	1.6	1.65	2.35
0.4	0.5	10	1.82	2.18	1.47	1.76	1.78	2.24

0.4	0.5	20	1.85	2.15	1.63	1.9	1.83	2.18
0.4	1	5	1.75	2.27	1.3	1.63	1.62	2.41
0.4	1	10	1.79	2.21	1.44	1.79	1.73	2.28
0.4	1	20	1.81	2.19	1.6	1.93	1.79	2.22
0.8	0.2	5	1.86	2.14	1.64	1.9	1.85	2.17
0.8	0.2	10	1.9	2.1	1.77	1.96	1.89	2.11
0.8	0.2	20	1.92	2.08	1.85	2	1.92	2.08
0.8	0.5	5	1.85	2.15	1.62	1.9	1.83	2.17
0.8	0.5	10	1.89	2.12	1.76	1.98	1.88	2.13
0.8	0.5	20	1.91	2.09	1.84	2.02	1.91	2.1
0.8	1	5	1.84	2.17	1.62	1.91	1.82	2.19
0.8	1	10	1.87	2.13	1.74	1.99	1.86	2.14
0.8	1	20	1.89	2.11	1.82	2.04	1.89	2.12

Table 25. Average Confidence Intervals about β_2 estimates in the large scale simulations for Probit Regression Contextual Effects Models

Simulation Parameters			Model A		Model B		Model C	
τ_{xx}	Ψ	Nj	Avg LL	Avg UL	Avg LL	Avg UL	Avg LL	Avg UL
0.1	0.2	5	1.45	2.54	0.82	1.18	-3.11	8.59
0.1	0.2	10	1.58	2.45	0.96	1.23	0.26	6.17
0.1	0.2	20	1.64	2.37	1.02	1.32	1.09	3.11
0.1	0.5	5	1.33	2.64	0.89	1.2	-7.06	14.15
0.1	0.5	10	1.44	2.56	0.92	1.26	-0.83	5.62
0.1	0.5	20	1.5	2.5	0.96	1.38	0.73	3.43
0.1	1	5	1.19	2.81	0.88	1.23	-24.3	32.89
0.1	1	10	1.28	2.73	0.87	1.31	-2.15	7.57
0.1	1	20	1.6	2.65	0.89	1.44	0.26	3.86
0.4	0.2	5	1.83	2.18	1.35	1.57	1.75	2.27
0.4	0.2	10	1.87	2.13	1.51	1.73	1.84	2.17
0.4	0.2	20	1.9	2.11	1.67	1.86	1.88	2.13
0.4	0.5	5	1.81	2.21	1.31	1.59	1.12	2.32
0.4	0.5	10	1.84	2.17	1.49	1.75	1.8	2.22
0.4	0.5	20	1.87	2.14	1.64	1.89	1.85	2.17
0.4	1	5	1.76	2.24	1.28	1.61	1.65	2.35
0.4	1	10	1.8	2.2	1.45	1.77	1.74	2.26
0.4	1	20	1.82	2.18	1.6	1.92	1.79	2.21
0.8	0.2	5	1.86	2.13	1.63	1.89	1.85	2.15
0.8	0.2	10	1.89	2.09	1.76	1.95	1.89	2.1
0.8	0.2	20	1.92	2.07	1.85	2	1.92	2.08
0.8	0.5	5	1.85	2.15	1.62	1.9	1.83	2.17
0.8	0.5	10	1.89	2.11	1.76	1.98	1.88	2.12
0.8	0.5	20	1.91	2.09	1.84	2.02	1.9	2.09
0.8	1	5	1.83	2.17	1.61	1.92	1.81	2.19
0.8	1	10	1.86	2.13	1.74	1.99	1.85	2.14

0.8	1	20	1.89	2.11	1.82	2.04	1.88	2.12

In this table it is seen that the model C with the EB mean (μ_j^{EB}) as the level-2 mean consistently has a higher coverage than the model B with the sample mean. The table also shows that the true model A consistently has a coverage percent at or around 95, which vindicates the 95% confidence interval. What varies, however, is the strength of the coverage as the ratios change. When both τ_{xx}/σ and τ_{xx}/Ψ are small (τ_{xx}/σ = 10% and τ_{xx}/Ψ < 50%) the coverage is not high for models B or C, and when they are high it is because the standard error of the slope estimate is very large, thus inflating the confidence interval length and therefore containing the true value more often (these cases specifically are marked with an asterisk). This coverage is not accurate and useful, and should be viewed with caution. As τ_{xx}/σ and τ_{xx}/Ψ become larger, Model C has coverage that is very high, and quite similarly performing to model A, in which the true values are known. Largely, these trends continue and the accuracy is better with larger group sizes (N_j) as well. Model B does not approach the coverage of either Models A or C, as it appears to produce inherently biased estimates of the between effect, thus biasing the contextual effect estimate. The performance of the sample mean as an estimate for the true mean (Model B) does improve with larger group sizes (N_j) as expected, as indicated by increased coverage as τ_{xx}/σ and τ_{xx}/Ψ become large. These trends are very similar for the Logit and Probit models, although the Probit models have more prevalence of cases where the coverage is high but the standard errors are large incorrectly inflating the coverage, which happens at N_j=10 for the small τ_{xx}/σ and τ_{xx}/Ψ.

Table 26. Results of large scale simulations, containing Coverage (%) for level-2 slope parameter.

	τ/Ψ (→)	% Coverage for models A/B/C - Logit							
Nj	τ/σ (↓)	10%	20%	40%	50%	80%	160%	200%	400%
5	10%	94.8/0/78.3	93.8/0/78.8		93.3/0/79.1				
10	10%	95.3/0/93.3*	96.2/0/92.4		95.5/0/90.9				
20	10%	95.6/0/95*	92.5/0/94.5*		95.3/0/94.2*				
5	40%			94.1/0.1/95.2		95.3/0/92.9		93.1/0/94	
10	40%			93.1/1.3/94.3		93.9/0.1/93.5		96/0/93.9	
20	40%			94.8/21.7/94.4		93.8/7.9/94		93.8/1.1/94.6	
5	80%					95.1/14.4/95.2	95.5/8.6/95.6		96/6.5/96.8
10	80%					95.4/42.1/94.8	94.4/33.9/94		95.5/23.5/94.9
20	80%					95.3/71.2/95.1	95.3/63.5/95.3		94.3/53.2/94.6
	τ/Ψ (→)	% Coverage for models A/B/C – Probit							
Nj	τ/σ (↓)	10%	20%	40%	50%	80%	160%	200%	400%
5	10%	94.4/0/77.5	95.4/0/77		93.9/0/74.4				
10	10%	94.5/0/90.3*	95.8/0/89.1*		95.1/0/88.9*				
20	10%	95.3/0.1/94.6*	95.5/0/93.3*		96.4/0/91.1*				
5	40%			95/0/92.7		95.2/0/92.7		94.6/5/91.8	
10	40%			95/0.3/94.7		94.5/0.1/93		94.3/0/94	
20	40%			95.2/18.2/95		94.6/4.5/95.1		96.4/0.3/95	
5	80%					95.1/19.2/93.6	94.8/12.1/94		94.6/5/94.2
10	80%					96.1/42.9/95.5	95.9/33.1/95.5		92.6/16.4/93.5
20	80%					95.3/74.2/95.2	93.6/62/94.5		91.9/46.7/95.5

* denotes that even though there is a high coverage %, the fact is that the coverage rate is inflated by a very large standard error of the estimate

3.3 Estimating the Effect of Duration of Adolescent Height Growth on Hypertension in Adulthood

3.3.1 Rationale

To accompany the results in the large-scale simulations, an example from the Fels Longitudinal Study data is carried out for a hands-on application of the findings. We have structured all of the findings to appear similarly to likened examples in literature. For example, Skrondal and Hesketh have been instrumental in the development of the GLLAMM package in STATA, and have presented findings in HGLM studies, including some with contextual effect modeling. In 2009, they published a paper including an analysis using HGLM for a logistic regression model set up identically setup to our simulations and used the sample mean ($\overline{X}_J$) for estimation of the contextual effect. Our results show that the estimate shown in the paper is a biased estimate of the true contextual effect in the population. In fact, for direct comparison our set-up is identical to theirs (as far as terminology, model structure, etc). This analysis and explanation we present is useful to show the differences using data in a biomedical analysis.

This study will be a two-level analysis with a dichotomous outcome, and both probit and logit models will be fit to it and the between effect and contextual effect of $\Delta_{[0,2]}$ will be studied for individuals within birth years. This is the same modeling setup that has been used in simulations thus far. Since the FLS has longitudinal blood pressure measurements, we can classify individuals as hypertensive if they have a diastolic blood pressure greater than 90mmHG or systolic blood pressure greater than 140mmHG. Then to create the two level analysis of individuals within birth years, we create an indicator variable of whether or not the individual was ever hypertensive in adulthood. This allows the fitting of the two-level model of individuals within birth years, with $\Delta_{[0,2]}$ calculated on individuals, and birth year mean $\Delta_{[0,2]}$ values at the second level of the analysis. Models with sample means and Empirical Bayes (EB) means will be fit and compared as they were in the large scale simulations. Setting up the outcome as "ever hypertensive" allows for direct comparison of results of this analysis of the Fels data to the results of both sets of simulations offered for logistic regression and probit regression models. This analysis would also make perfect sense as a 3-level analysis with longitudinal hypertension indicator varying occasion to occasion (as blood pressure is measured in adulthood in the FLS) and those occasions nested within individuals within birth years. However since the studies to this point are only for a 2-level HGLM, we maintain that structure.

3.3.2 Data Description

We have a possible 558 men and 608 women in the FLS who have the childhood height measurements adequate for measurement of $\Delta_{[0,2]}$ and who also have adulthood blood pressure measurements. Out of these 31.4% of men have had a hypertensive measurement, and 18.2% of women have had a hypertensive measurement. This rate of prevalence of hypertension in the FLS should allow for accurate estimation of the binary response models.

To compare this example to the simulation results, it will be important to compare $\Delta_{[0,2]}$ to the X variable simulated in our studies. That is, X will vary at both levels of this analysis, the person-to-person level and the birth-year level. The following shows the form of $\Delta_{[0,2]}$:

$$\Delta_{[0,2]jk} = \mu_{[0,2]} + u_k + \varepsilon_{jk}$$

where, for individual j in birth year k, we have the following for the above model:

$$\mu_{[0,2]} = 0.44$$

$$u_k \sim N\left(0, \tau_{[0,2]}\right);\ \tau_{[0,2]} = 0.13$$

$$\varepsilon_{jk} \sim N\left(0, \sigma^2_{[0,2]}\right); \sigma^2_{[0,2]} = 0.52$$

The birth years have an average of 20 people belonging to them (N_j in the simulations). Thus the comparable situation in the simulations is N_j=20, τ_{xx}=0.2 (σ=1), and Ψ as shown in the model tables in this section. However, τ should also be regarded as a percentage of the level-2 variance components estimated in the models and compared to the τ_{xx}/Ψ ratios as appropriate when regarding the model parameter estimates and the standard errors of those estimates.

3.3.3 Results

The two sets of models we fit are the following for an individual j in birth years k:

Logit Models:

$$logit\left(HyperT_{jk}\right) = \beta_0 + \beta_F * Female_{jk} + \beta_1 * \left(\Delta_{[0,2]jk} - \overline{\Delta_{[0,2]k}}\right) + \beta_2 * \left(\overline{\Delta_{[0,2]k}}\right) + u_k$$

$$logit\left(HyperT_{jk}\right) = \beta_0 + \beta_F * Female_{jk} + \beta_1 * \left(\Delta_{[0,2]jk} - \mu^{EB}_{[0,2]k}\right) + \beta_2 * (\mu^{EB}_{[0,2]k}) + u_k$$

Probit Models:

$$Probit\left(HyperT_{jk}\right) = \beta_0 + \beta_F * Female_{jk} + \beta_1 * \left(\Delta_{[0,2]jk} - \overline{\Delta_{[0,2]k}}\right) + \beta_2 * \left(\overline{\Delta_{[0,2]k}}\right) + u_k$$

$$Probit\left(HyperT_{jk}\right) = \beta_0 + \beta_F * Female_{jk} + \beta_1 * \left(\Delta_{[0,2]jk} - \mu^{EB}_{[0,2]k}\right) + \beta_2 * (\mu^{EB}_{[0,2]k}) + u_k$$

where we fit the within effect of $\Delta_{[0,2]}$, the between effect of $\Delta_{[0,2]}$ and take into account gender differences in hypertension prevalence. In both the Logit and the Probit sets, the first model contains the sample mean of $\Delta_{[0,2]jk}$ as calculated from the data, denoted $\overline{\Delta_{[0,2]k}}$ for each birth year k. The second model is the EB model we discuss in previous sections where the Empirical Bayes estimate of the true birth year mean of $\Delta_{[0,2]jk}$ is used, denoted $\mu^{EB}_{[0,2]k}$ for each birth year k. Obviously, we cannot fit the true (known) model as shown in the simulations because we do not know the true means. These models that we have fit to this point are referred to as Unit Specific models (versus Population Average, Zeger et. al. 1988). As described by Zeger et al the distinction is in the estimation of the random effect, and whether it is a conditional or marginal model. Our intention is to estimate the random effect for each birth year as well as the slope parameters, thus the unit specific methods are more applicable. The results for the two sets of models we fit to compare results of sample means and Empirical Bayes means as estimates for true group means are in the following table.

Table 27. Results of Hypertension Example Models

Logit Models								
Sample Mean Model					EB Mean Model			
Term	Estimate	S.E.	p-value		Term	Estimate	S.E.	p-value
Intercept	0.45	0.32	0.162		Intercept	10.09	2.29	<.0001
Female	-0.93	0.07	<.0001		Female	-0.93	0.07	<.0001
$\Delta_{[0,2]}$	0.32	0.05	<.0001		$\Delta_{[0,2]}$	0.32	0.05	<.0001
$\overline{\Delta_{[0,2]}}$	-1.78	0.43	<.0001		$\mu^{EB}_{[0,2]}$	-5.34	1.17	<.0001
Random Int (Ψ)	0.89	0.21			Random Int (Ψ)	0.89	0.21	
For Reference: τ_{xx} / Ψ = 0.13/0.89 = 0.15; τ_{xx} / σ = 0.13 / 0.52 = 0.25								
Probit models								
Sample Mean Model					EB Mean Model			
Term	Estimate	S.E.	p-value		Term	Estimate	S.E.	p-value
Intercept	0.24	0.18	0.196		Intercept	5.72	1.30	<.0001
Female	-0.55	0.04	<.0001		Female	-0.55	0.04	<.0001
$\Delta_{[0,2]}$	0.18	0.03	<.0001		$\Delta_{[0,2]}$	0.18	0.03	<.0001
$\overline{\Delta_{[0,2]}}$	-1.01	0.25	<.0001		$\mu^{EB}_{[0,2]}$	-3.03	0.67	<.0001
Random Int (Ψ)	0.30	0.07			Random Int (Ψ)	0.30	0.07	
For Reference: τ_{xx} / Ψ = 0.13/0.30 = 0.43; τ_{xx} / σ = 0.13 / 0.52 = 0.25								

3.3.4 Comparison of Results to Simulation Studies

The table above offers a clear comparison to the simulations previously presented. We can compare estimates of the slopes and their standard errors and use the previously presented simulations for commentary on the differences shown. In both the Logit and the Probit models, we see the level one slope coefficients (Female and $\Delta_{[0,2]}$) remaining the same in both models (Sample mean and EB) along with the estimate of the second level variance component, as expected. The intercept does change, and the between birth-years effect of $\Delta_{[0,2]}$ is larger in magnitude for the EB model (versus the sample mean model) in both methods of analyzing the binary outcome.

In terms of the relationship τ_{xx} and Ψ, we see that in the Logit analysis, τ is 14.6% of Ψ (43% for probit), and the standard error for the between effect estimate is much larger for the EB model than in the sample mean model, which we have noted should lead to cautious interpretation of results. However, with N_j=20 on average, the EB model is likely to be stable enough for analysis (inferences seem satisfactory based on the simulation results with N_j=20). The ratio of τ_{xx} and σ of 0.25 is large enough, with the group sizes, to also give reliable EB estimates and with the caveat of the larger standard errors, the EB model can be used with some confidence from the effect sizes much larger than the standard errors. In the smaller scale (Ch 2, Tables 10-12, 13-15) simulations there are approximations close to this example's τ_{xx}/Ψ and τ_{xx}/σ ratios is shown with one case (σ=1, τ_{xx}=0.1 or 0.2, Ψ=0.5 or 1, N_j=20) and with the larger group sizes the estimates of the parameters are closer to the true model in the EB model than the sample mean model estimates, and the standard errors are not large enough to raise questions about the inaccuracy of the estimates. Though those exact situations weren't all simulated repeatedly over many times in this chapter, the results from the larger scale simulations are likely to apply. Limitations of this analysis will be explained thoroughly in the next few sections as extensions are discussed.

3.3.5 Interpretation of Results

In terms of biological plausibility to this model, we know from literature that childhood obesity trends affect the likelihood of hypertension in adulthood (Sun et al 2008). We see that $\Delta_{[0,2]}$ is significantly positively associated with adulthood hypertension, much as it was with BMI. That is, the longer an individual spends growing little in adolescence, the higher chance that individual will have hypertension in adulthood. In the logistic regression model, this is seen by looking at odds ratios, and for both of these models the link function can be used to predict a probability of an individual having hypertension in adulthood. Thus, the extended length of either slow or no growth in adolescence increases the probability (or odds) of hypertension in adulthood. The model also shows that there is less chance of hypertension for females than males, and that generational differences in adolescent height growth impact hypertension as well as BMI. This may be biased however, as age of hypertension and other important factors are not taken into account. This example serves as a good illustration of methods, but also points out that linking adulthood hypertension and adolescent growth behavior may be of interest in the future. Further expansion on this idea will be contained in the future work section, where plausible extensions can be more thoroughly discussed.

It is also quite known that metabolic syndrome is an attempt at reconciling the relationship between BMI, blood pressure (hence, hypertension), and blood lipid profiles (Sun et al 2007). We have shown that the amount of time spent growing slowly (or not growing) in adolescence is related to adulthood obesity, and thus, we may find positive association between $\Delta_{[0,2]}$ and likelihood of having hypertension in adulthood in a similar way. Perhaps the more interesting point, then, is that the significant secular trend exists in hypertension, much in the same way as secular trend in BMI was seen previously. This is indicated by the largely significant between birth-year effect (thus contextual effect) of the $\Delta_{[0,2]}$ on susceptibility to having hypertension in adulthood. The model analyzed in this section illustrates the use of the simulation results in analyzing hierarchical nonlinear models with a contextual effect. However, it is limited in the sense that it does not control for an extensive list of covariates, which I discuss in the next section.

3.4 Possible Extensions

This dissertation explores normally distributed predictor variables on normal outcomes as well as non-normal outcomes that require estimation of a non-linear model or HGLM. Further considerations could be whether or not the predictor variable is truly normally distributed. That is, if the predictor is skewed or discrete, then these study result may not hold as true. It may be of interest to explore these possibilities. This could be explored for outcome variables with different distributions, as well.

The data in the FLS, specifically Δ as shown in chapter 1, has enough higher level variability so that the theory of using the EB mean as shown in Shin and Raudenbush (2010) is applicable (as explained here in simulations and discussion). They state that as the group sample sizes increase, the estimates improve. However, it is not as well defined how the estimates act with small sample sizes, and with small cluster-level group variability. We have seen with small higher-level variability in the predictor X, in which the contextual effect of X is sought, the estimate for the contextual effect is not well estimated with the EB method or the sample mean method, as standard errors become large and estimates become biased. This phenomenon is explored more as simulations are run for outcomes that are not normally distributed in chapters two and three. It should be noted that if the predictor variable does not vary at the higher level, it does not make sense to seek a contextual effect of that variable.

Our estimation in the reported study has been carried out by maximum likelihood. Biased estimates or large standard errors we have seen with small sample sizes may be improved by Bayesian estimation. Further examination of these situations by using a Bayesian analysis with Markov Chain Monte Carlo (MCMC) methods may help to show the behavior of the parameters in these conditions. This future endeavor could help researchers know which methodology to use in the small sample size estimation. The Bayesian methods are not summarized in this dissertation.

Another extension that could potentially solidify the results shown in this paper is the use of missing data analysis in the HLM2, HGLM2, and HLM3 analyses shown in this thesis. The analyses completed were done under complete case analysis. There could be efficiency gain in imputing any individuals with missing data under the Missing At Random (MAR) assumption (Ruben 1976). Using such methods may produce more efficient estimation of the effects studied above (Shin and Raudenbush 2007).

Lastly, the brief introduction into the study of hypertension was displayed as a means of illustrating the usefulness of our simulation results. In that way, the models terms in the model were simple and merely used for the illustrative purpose. However, this analysis did show the possibility of a true association between individual adolescent growth duration and adulthood hypertension. Before this is fully established several other factors must be accounted for. For one, an individual's age at first hypertensive visit or mean age in this sample would be useful. Hypertension worsens with age, and if someone in our sample only had visits in their 20s, then their likelihood of hypertension is likely already lessened.

Also, the definition of hypertension is debated. That is, the cutoffs used of DBP$\geq$90mmHg and SBP$\geq$140mmHg are not gold standard cutoffs, and some people use other values as cutoffs. It is also sometimes noted that one measurement higher than the defined cutoffs may not be enough to define and individual as hypertensive. This point is directly related to another point that is an obvious limitation of this analysis. The outcome variable of hypertension yes/no is useful for our illustration of binary data analysis, but is not entirely using all of the information from the FLS. Serial blood pressure measurements are recorded at regular examinations, and for the example shown we summarize all adulthood visits by a single dichotomous value. We may be ignoring age trends (as previously mentioned) but also serial variability in blood pressure,.

The FLS records whether or not individuals are on blood pressure medication, and not accounting for that could inherently bias the results. The Fels study also includes disease and pregnancy statuses that could be screened for a study of this kind that could affect results. When a final model is chosen, a simulation could be run directly fitting the data structure of the hypertension example and the use of the sample mean and EB mean for the contextual effect could be compared in this specific case (much like Table 9 in Ch1). This could strengthen the claim that the EB method lends a closer approximation to the true contextual effect. What is hoped to be shown in this section is not the glaring limitations of our example, but rather the possibilities of extending this to a novel discussion on the link between adolescent growth behavior and adulthood blood pressure variability, including hypertension.

3.5 Description of Appendices

Accompanying this chapter are the results of these simulations, whose results are described verbally. We attach the SAS code used for simulating the data, fitting the models, and repeating the process m times. The SAS code will also provide calculation of coverage percentages as well as summary measures of the parameters. The tabled results of the simulations run are also contained in an appendix. Using the tables, values of the parameters estimated in each model can be seen as well as an idea of the variability in these estimates. All SAS programs are written and executed in version 9.3.

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Appendix A

SAS Code for Simulations in Chapter 2

This appendix shows the SAS code used for the simulations of all five different generalized linear models used in Chapter 2. In order, Logit (logistic regression), Probit, Poisson (log-linear), Ordinal regression (generalized probit), and Nominal regression are shown. The code is presented in macros, with a description of macro variables for use when running the code.

```
/**** LOGIT SIM ****/
/* Macro variables are as follows:
        IT - a counter for running of multiple iterations, but saving different data sets as needed
        NJ - number of observations in each of the 1000 groups
        TAO - level 2 variability of explanatory variable X
        PSI - level 2 random error for modeling
*/
%macro LOGIT(IT, NJ, TAO, PSI);
data sim&IT;
        do i=1 to 1000;                                         /* 1000 second level groups
*/
                u&IT=rand('NORMAL', 0, &tao);    /* tao and psi */
                ps&IT=rand('NORMAL', 0, sqrt(&PSI));
                do j=1 to &NJ;                                  /* nj in each group */
                        er&IT=rand('NORMAL', 0, 1); output;     /* fix sigma to 1 */
                end;
        end;
run;

data sim&IT; set sim&IT;
        d&IT=u&IT+er&IT;                                /* group observed values, u's are
"true means" */
        NN=_n_;                                                         /* ers are level 1
scaled d's */
run;
proc means data=sim&IT noprint;
        by i; var d&IT;
        output out=MEANS;
run;
data means; set means;
        if _STAT_="MEAN";
        dbar&IT=d&IT;                                           /* group sample means */
keep i dbar&IT;
run;

data sim&IT; merge sim&IT means; by i;
        keep i j u&IT ebar&IT er&IT ps&IT d&IT dbar&IT nn;
        ebar&IT= d&IT-dbar&IT;
run;
```

```
data sim&IT; set sim&IT;
        linp&IT = 1 + 1*er&IT + 2*u&IT +ps&IT;            /* linear predictor */
run;
data sim&IT; set sim&IT;
        prob&IT = exp(linp&IT)/(1+exp(linp&IT)) ;/* level one variability */
        yhat&IT = rand('BERNOULLI', prob&IT);             /* yhat for sims */
run;
data sim&IT; set sim&IT;

proc sort data=sim&IT; by yhat&IT descending ; run;
/* we output and save data sets for estimates */
ods output ParameterEstimates=trueParams&IT;
ods output CovParms=trueCovParms&IT;
proc glimmix data=sim&IT;                                   /* known model A */
        class i j;
        model yhat&IT(event=LAST) = u&IT er&IT / s dist=bernoulli link=logit ;
        random i ;
run;
ods output close;

ods output ParameterEstimates=barParams&IT;
ods output CovParms=barCovParms&IT;
proc glimmix data=sim&IT;                                   /* sample mean model B */
        class i j;
        model yhat&IT(event=LAST) = dbar&IT ebar&IT / s dist=bernoulli link=logit ;
        random i ;
run;
ods output close;
proc append base=trueparams&IT data=truecovparms&IT force; run;
proc append base=barparams&IT data=barcovparms&IT force; run;

ods output CovParms=CovParms&IT;
ods output ParameterEstimates=Params&IT;
proc glimmix data=sim&IT;                                   /* the following calculates
reliability */
        class i;                                                    /* and EB group mean
estimates */
        model d&IT = / s ;
        random i;
run;

data ebset1; set covparms&IT;
        do i=1 to &nj*1000;
                where covparm="i"; do; tao=estimate; end;
                        output;
        end;
```

```
        keep tao;
run;
data ebset2; set covparms&IT;
        do i=1 to &nj*1000;
                where covparm="Residual"; do; sig=estimate; end;
                 output;
        end;
        keep sig;
run;
data ebset3; set params&IT;
        do i=1 to &nj*1000;
        int=estimate;
                 output;
        end;
        keep int;
run;
data ebset; merge ebset1 ebset2 ebset3; run;
data eb; merge ebset sim&IT; run;
data eb; set eb;
        rel= tao / (tao + (sig/ &nj ));              /* reliability as nj constant */
run;
data eb; set eb;
        deb&IT= (1-rel)*INT + rel*dbar&IT;            /* EB Estimate of mean */
run;
data eb; set eb;
        eeb&IT= d&IT-deb&IT;
run;

ods output ParameterEstimates=ebParams&IT;
ods output CovParms=ebCovParms&IT;
proc glimmix data=eb method=gauss;                                   /* fit EB model C */
        class i j;
        model yhat&IT(event=LAST) = deb&IT eeb&IT / s dist=bernoulli link=logit ;
        random intercept / subject= i ;
run;
ods output close;
proc append base=ebparmsrun&RUN data=ebparams&IT force; run;
proc append base=ebparmsrun&RUN data=ebcovparms&IT force; run;
%mend;
%LOGIT(1, 10, .4, 0.5); /* example to run, with each below, too */
```

```
/**** PROBIT ****/
%macro PROBIT(IT, NJ, TAO, PSI);
data sim&IT;
	do i=1 to 1000;
		u&IT=rand('NORMAL', 0, &tao);
		ps&IT=rand('NORMAL', 0, sqrt(&PSI));
		do j=1 to &NJ;
			er&IT=rand('NORMAL', 0, 1); output;
		end;
	end;
run;

data sim&IT; set sim&IT;
	d&IT=u&IT+er&IT;
	NN=_n_;
run;
proc means data=sim&IT noprint;
	by i; var d&IT;
	output out=MEANS;
run;
data means; set means;
	if _STAT_="MEAN";
	dbar&IT=d&IT;
keep i dbar&IT;
run;

data sim&IT; merge sim&IT means; by i;
	keep i j ebar&IT u&IT er&IT ps&IT d&IT dbar&IT nn;
	ebar&IT= d&IT-dbar&IT;
run;

data sim&IT; set sim&IT;
	prolinp&IT = 1 + 1*er&IT + 2*u&IT +ps&IT;
run;
data sim&IT; set sim&IT;
	proprob&IT = cdf('NORMAL', prolinp&IT, 0, 1) ;
	proyhat&IT = rand('BERNOULLI', proprob&IT);
run;
data sim&IT; set sim&IT;
proc sort data=sim&IT; by yhat&IT descending ; run;
ods output ParameterEstimates=trueParams&IT;
ods output CovParms=trueCovParms&IT;
proc glimmix data=sim&IT method=gauss;
	class i;
	model proyhat&IT(event=LAST) = u&IT er&IT / s dist=binary link=probit  ;
	random int / subject=i ;
```

```
run;
ods output close;

ods output ParameterEstimates=barParams&IT;
ods output CovParms=barCovParms&IT;
proc glimmix data=sim&IT method=gauss;
        class i;
        model proyhat&IT(event=LAST) = dbar&IT ebar&IT / s dist=binary link=probit ;
        random int / subject=i ;
run;
ods output close;
proc append base=trueparams&IT data=truecovparms&IT force; run;
proc append base=barparams&IT data=barcovparms&IT force; run;
proc print data=trueparams&IT; run;
proc print data=barparams&IT; run;

ods output CovParms=CovParms&IT;
ods output ParameterEstimates=Params&IT;
proc glimmix data=sim&IT;
        class i;
        model d&IT = / s ;
        random i;
run;

data ebset1; set covparms&IT;
        do i=1 to &nj*1000;
                where covparm="i"; do; tao=estimate; end;
                        output;
        end;
        keep tao;
run;
data ebset2; set covparms&IT;
        do i=1 to &nj*1000;
                where covparm="Residual"; do; sig=estimate; end;
                 output;
        end;
        keep sig;
run;
data ebset3; set params&IT;
        do i=1 to &nj*1000;
        int=estimate;
                 output;
        end;
        keep int;
run;
data ebset; merge ebset1 ebset2 ebset3; run;
```

```
data eb; merge ebset sim&IT; run;
data eb; set eb;
	rel= tao / (tao + (sig/ &nj ));
run;
data eb; set eb;
	deb&IT= (1-rel)*INT + rel*dbar&IT;
run;
data eb; set eb;
	eeb&IT= d&IT-deb&IT;
run;

ods output ParameterEstimates=ebParams&IT;
ods output CovParms=ebCovParms&IT;
proc glimmix data=eb method=gauss;
	class i j;
	model proyhat&IT(event=LAST) = deb&IT eeb&IT / s dist=binary link=probit ;
	random intercept / subject= i ;
run;
ods output close;
proc append base=ebparmsrun&RUN data=ebparams&IT force; run;
proc append base=ebparmsrun&RUN data=ebcovparms&IT force; run;

%mend;
%PROBIT(1, 10, .4, 0.5);

/**** POISSON ****/
%macro poisson(IT, NJ, TAO, PSI);
data sim&IT;
	do i=1 to 1000;
		u&IT=rand('NORMAL', 0, &tao);
		ps&IT=rand('NORMAL', 0, sqrt(&PSI));
		do j=1 to &NJ;
			er&IT=rand('NORMAL', 0, 1); output;
		end;
	end;
run;

data sim&IT; set sim&IT;
	d&IT=u&IT+er&IT;
	NN=_n_;
run;
proc means data=sim&IT noprint;
	by i; var d&IT;
	output out=MEANS;
run;
data means; set means;
```

```
        if _STAT_="MEAN";
        dbar&IT=d&IT;
keep i dbar&IT;
run;

data sim&IT; merge sim&IT means; by i;
        keep i j u&IT ebar&IT er&IT ps&IT d&IT dbar&IT nn;
        ebar&IT= d&IT-dbar&IT;
run;
data sim&IT; set sim&IT;
        linp&IT = 1 + 1*er&IT + 2*u&IT +ps&IT;
run;
data sim&IT; set sim&IT;
        poismean&IT = exp( linp&IT) ;
        poisyhat&IT = rand('POISSON', poismean&IT);
run;
data sim&IT; set sim&IT;
ods output ParameterEstimates=trueParams&IT;
ods output CovParms=trueCovParms&IT;
proc glimmix data=sim&IT method=gauss;
        class i;
        model poisyhat&IT = u&IT er&IT / s dist=poisson link=log  ;
        random int / subject=i ;
run;
ods output close;

ods output ParameterEstimates=barParams&IT;
ods output CovParms=barCovParms&IT;
proc glimmix data=sim&IT method=gauss;
        class i;
        model poisyhat&IT = dbar&IT ebar&IT / s dist=poisson link=log ;
        random int / subject=i ;
run;
ods output close;
proc append base=trueparams&IT data=truecovparms&IT force; run;
proc append base=barparams&IT data=barcovparms&IT force; run;
proc print data=trueparams&IT; run;
proc print data=barparams&IT; run;

ods output CovParms=CovParms&IT;
ods output ParameterEstimates=Params&IT;
proc glimmix data=sim&IT;
        class i;
        model d&IT = / s ;
        random i;
run;
```

```
data ebset1; set covparms&IT;
	do i=1 to &nj*1000;
		where covparm="i"; do; tao=estimate; end;
			output;
	end;
	keep tao;
run;
data ebset2; set covparms&IT;
	do i=1 to &nj*1000;
		where covparm="Residual"; do; sig=estimate; end;
		 output;
	end;
	keep sig;
run;
data ebset3; set params&IT;
	do i=1 to &nj*1000;
	int=estimate;
		 output;
	end;
	keep int;
run;
data ebset; merge ebset1 ebset2 ebset3; run;
data eb; merge ebset sim&IT; run;
data eb; set eb;
	rel= tao / (tao + (sig/ &nj ));
run;
data eb; set eb;
	deb&IT= (1-rel)*INT + rel*dbar&IT;
run;
data eb; set eb;
	eeb&IT= d&IT-deb&IT;
run;

ods output ParameterEstimates=ebParams&IT;
ods output CovParms=ebCovParms&IT;
proc glimmix data=eb method=gauss;
	class i j;
	model poisyhat&IT  = deb&IT eeb&IT / s dist=poisson link=log ;
	random intercept / subject= i ;
run;
ods output close;
proc append base=ebparmsrun&RUN data=ebparams&IT force; run;
proc append base=ebparmsrun&RUN data=ebcovparms&IT force; run;
%mend;
%POISSON(1, 10, .4, 0.5);
```

```
/**** ORDINAL ****/
%macro ORDINAL(IT, NJ, TAO, PSI);
data sim&IT;
	do i=1 to 1000;
		u&IT=rand('NORMAL', 0, &tao);
		ps&IT=rand('NORMAL', 0, sqrt(&PSI));
		do j=1 to &NJ;
			er&IT=rand('NORMAL', 0, 1); output;
		end;
	end;
run;

data sim&IT; set sim&IT;
	d&IT=u&IT+er&IT;
	NN=_n_;
run;
proc means data=sim&IT noprint;
	by i; var d&IT;
	output out=MEANS;
run;
data means; set means;
	if _STAT_="MEAN";
	dbar&IT=d&IT;
keep i dbar&IT;
run;

data sim&IT; merge sim&IT means; by i;
	keep i j u&IT ebar&IT er&IT ps&IT d&IT dbar&IT nn;
		ebar&IT= d&IT-dbar&IT;
run;

data sim&IT; set sim&IT;
	LIJ&IT =1*( 1*er&IT + 2*u&IT + ps&IT + rand('Normal',0,1) );
run;

data sim&IT; set sim&IT;
	if LIJ&IT lt -1 then do;
		ygroup&IT=0; end;
	if LIJ&IT gt -1 and LIJ&IT le 2 then do;
		ygroup&IT=1; end;
	if LIJ&IT gt 2 then do;
		ygroup&IT=2; end;
run;
ods output ParameterEstimates=trueParams&IT;
ods output CovParms=trueCovParms&IT;
proc glimmix data=sim&it method=gauss;
```

```
        class i;
        model ygroup&it (descending)= er&it u&it / s link=cprobit dist=multinomial;
        random intercept / subject=i;
run;
ods output close;

ods output ParameterEstimates=barParams&IT;
ods output CovParms=barCovParms&IT;
proc glimmix data=sim&it method=gauss;
        class i;
        model ygroup&it (descending)= ebar&it dbar&it / s link=cprobit dist=multinomial;
        random intercept / subject=i;
run;
ods output close;
proc append base=trueparams&IT data=truecovparms&IT force; run;
proc append base=barparams&IT data=barcovparms&IT force; run;

ods output CovParms=CovParms&IT;
ods output ParameterEstimates=Params&IT;
proc glimmix data=sim&IT;
        class i;
        model d&IT = / s ;
        random i;
run;

data ebset1; set covparms&IT;
        do i=1 to &nj*1000;
                where covparm="i"; do; tao=estimate; end;
                        output;
        end;
        keep tao;
run;
data ebset2; set covparms&IT;
        do i=1 to &nj*1000;
                where covparm="Residual"; do; sig=estimate; end;
                 output;
        end;
        keep sig;
run;
data ebset3; set params&IT;
        do i=1 to &nj*1000;
        int=estimate;
                 output;
        end;
        keep int;
run;
```

```
data ebset; merge ebset1 ebset2 ebset3; run;
data eb; merge ebset sim&IT; run;
data eb; set eb;
	rel= tao / (tao + (sig/ &nj ));
run;
data eb; set eb;
	deb&IT= (1-rel)*INT + rel*dbar&IT;
run;
data eb; set eb;
	eeb&IT= d&IT-deb&IT;
run;

ods output ParameterEstimates=ebParams&IT;
ods output CovParms=ebCovParms&IT;
proc glimmix data=eb method=gauss;
	class i j;
	model ygroup&it (descending) = deb&IT eeb&IT / s dist=multinomial link=cprobit ;
	random intercept / subject= i ;
run;
ods output close;
proc append base=ebparmsrun&RUN data=ebparams&IT force; run;
proc append base=ebparmsrun&RUN data=ebcovparms&IT force; run;
%mend;
%ORDINAL(1, 10, .4, 0.5);


/**** Nominal ****/
%macro NOMINAL(IT, NJ, TAO, PSI1, PSI2);
data sim&IT;
	do i=1 to 1000;
		u&IT=rand('NORMAL', 0, &tao);
		do j=1 to &NJ;
			er&IT=rand('NORMAL', 0, 1); output;
		end;
	end;
run;

data sim&IT; set sim&IT;
	d&IT=u&IT+er&IT;
	NN=_n_;
run;
proc means data=sim&IT noprint;
	by i; var d&IT;
	output out=MEANS;
run;
data means; set means;
```

```
        if _STAT_="MEAN";
        dbar&IT=d&IT;
keep i dbar&IT;
run;

data sim&IT; merge sim&IT means; by i;
        keep i j u&IT ebar&IT er&IT d&IT dbar&IT nn;
                ebar&IT= d&IT-dbar&IT;
run;

data sim2&IT; set sim&IT;
        by i; if first.i;
        ps1&IT = Rand('NORMAL', 0, &PSI1);
        ps2&IT = Rand('NORMAL', 0, &PSI2);
        keep i ps1&IT ps2&IT;
run;
data sim&IT; merge sim&IT sim2&IT; by i; run;

data sim&IT; set sim&IT;
        e0= exp(1 + 1*er&IT + 2*u&IT + ps1&IT) ;
        e1= exp(2 + 3*er&IT + 5*u&IT + ps2&IT);
run;

data sim&IT; set sim&IT;
        p0= e0 / (1+ e0 + e1);
        p1= e1 / (1+ e0 + e1);
        p2= 1 - p0 - p1;
run;
data sim&IT; set sim&IT;
        ymn&IT = rand('TABLE', p0, p1, p2);
run;

proc sort data=sim&IT; by ymn&IT; run;
ods output ParameterEstimates=trueParams&nIT;
ods output CovParms=trueCovParms&IT;
proc glimmix data=sim&IT method=gauss;
        class i ymn&IT;
        model ymn&IT (ref=last)= er&IT u&it / s link=glogit dist=multinomial;
        random intercept / subject=i group=ymn&IT ;
run;
ods output close;

ods output ParameterEstimates=barParams&IT;
ods output CovParms=barCovParms&IT;
proc glimmix data=sim&it method=gauss;
        class i ymn&iT;
```

```
        model ymn&it (ref=last)= ebar&it dbar&it / s link=glogit dist=multinomial;
        random intercept / subject=i group=ymn&IT;
run;
ods output close;
proc append base=trueparams&IT data=truecovparms&IT force; run;
proc append base=barparams&IT data=barcovparms&IT force; run;

ods output CovParms=CovParms&IT;
ods output ParameterEstimates=Params&IT;
proc glimmix data=sim&IT;
        class i;
        model d&IT = / s ;
        random i;
run;

data ebset1; set covparms&IT;
        do i=1 to &nj*1000;
                where covparm="i"; do; tao=estimate; end;
                        output;
        end;
        keep tao;
run;
data ebset2; set covparms&IT;
        do i=1 to &nj*1000;
                where covparm="Residual"; do; sig=estimate; end;
                 output;
        end;
        keep sig;
run;
data ebset3; set params&IT;
        do i=1 to &nj*1000;
        int=estimate;
                 output;
        end;
        keep int;
run;
data ebset; merge ebset1 ebset2 ebset3; run;
data eb; merge ebset sim&IT; run;
data eb; set eb;
        rel= tao / (tao + (sig/ &nj ));
run;
data eb; set eb;
        deb&IT= (1-rel)*INT + rel*dbar&IT;
run;
data eb; set eb;
        eeb&IT= d&IT-deb&IT;
```

```
run;

ods output ParameterEstimates=ebParams&IT;
ods output CovParms=ebCovParms&IT;
proc glimmix data=eb method=gauss;
        class i j;
        model ymn&IT (ref=last) = deb&IT eeb&IT / s dist=multinomial link=glogit ;
        random intercept / subject= i group=ymn&IT;
run;
ods output close;
proc append base=ebparmsrun&RUN data=ebparams&IT force; run;
proc append base=ebparmsrun&RUN data=ebcovparms&IT force; run;
%mend;
%NOMINAL(1, 10, .4, 0.5, 1);
```

Appendix B

Tabled Results for Simulations in Chapter 2

Appendix – Accompanying tables to Simulations for Chapter Two

This appendix shows the results of the simulations in all five generalized linear models explored in Chapter 2. In the same order as shown in the paper, each model has a table for each value of Ψ, and each table is sorted by model parameter, then group sample size, then τ_{xx}.

1. Logit Simulations

Table B.1. Logit, Ψ=0.2

Ψ=0.2	Model A					Model B					Model C			
	Estimate	SE	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj
B0	0.98	0.04	0.1	5	B0	0.98	0.04	0.1	5	B0	0.97	0.04	0.1	5
B0	0.91	0.04	0.2	5	B0	0.92	0.04	0.2	5	B0	0.92	0.01	0.2	5
B0	0.89	0.04	0.3	5	B0	0.89	0.04	0.3	5	B0	0.96	0.01	0.3	5
B0	0.96	0.04	0.4	5	B0	0.96	0.04	0.4	5	B0	0.95	0.04	0.4	5
B0	1.05	0.04	0.8	5	B0	1.06	0.04	0.8	5	B0	1.06	0.04	0.8	5
B0	0.97	0.03	0.1	10	B0	0.97	0.03	0.1	10	B0	0.94	0.03	0.1	10
B0	1.01	0.03	0.2	10	B0	1.01	0.03	0.2	10	B0	0.96	0.00	0.2	10
B0	0.98	0.03	0.3	10	B0	0.99	0.03	0.3	10	B0	0.98	0.01	0.3	10
B0	0.94	0.03	0.4	10	B0	0.94	0.03	0.4	10	B0	0.93	0.03	0.4	10
B0	0.93	0.03	0.8	10	B0	0.92	0.03	0.8	10	B0	0.91	0.03	0.8	10
B0	1.00	0.02	0.1	20	B0	0.99	0.02	0.1	20	B0	1.00	0.02	0.1	20
B0	0.95	0.02	0.2	20	B0	0.95	0.02	0.2	20	B0	0.95	0.00	0.2	20
B0	0.95	0.02	0.3	20	B0	0.95	0.02	0.3	20	B0	0.94	0.00	0.3	20
B0	0.95	0.02	0.4	20	B0	0.95	0.02	0.4	20	B0	0.95	0.02	0.4	20
B0	0.96	0.03	0.8	20	B0	0.96	0.03	0.8	20	B0	0.96	0.03	0.8	20
B0	1.02	0.02	0.1	40	B0	1.01	0.02	0.1	40	B0	0.98	0.00	0.1	40
B0	0.99	0.02	0.2	40	B0	0.99	0.02	0.2	40	B0	0.96	0.00	0.2	40
B0	1.00	0.02	0.3	40	B0	1.00	0.02	0.3	40	B0	0.96	0.00	0.3	40
B0	0.99	0.02	0.4	40	B0	0.99	0.02	0.4	40	B0	0.98	0.00	0.4	40
B0	0.97	0.02	0.8	40	B0	0.97	0.02	0.8	40	B0	0.96	0.00	0.8	40
B0	0.98	0.02	0.1	80	B0	0.98	0.02	0.1	80	B0	0.95	0.00	0.1	80
B0	1.01	0.02	0.2	80	B0	1.01	0.02	0.2	80	B0	0.99	0.00	0.2	80

B0	0.97	0.02	0.3	80	B0	0.97	0.02	0.3	80	B0	0.94	0.00	0.3	80
B0	0.96	0.02	0.4	80	B0	0.96	0.02	0.4	80	B0	0.96	0.00	0.4	80
B0	0.96	0.02	0.8	80	B0	0.95	0.02	0.8	80	B0	0.94	0.00	0.8	80
B1	0.96	0.04	0.1	5	B1	0.95	0.04	0.1	5	B1	0.95	0.04	0.1	5
B1	0.94	0.04	0.2	5	B1	0.92	0.04	0.2	5	B1	0.96	0.01	0.2	5
B1	0.95	0.04	0.3	5	B1	0.94	0.04	0.3	5	B1	0.95	0.01	0.3	5
B1	1.00	0.04	0.4	5	B1	1.00	0.04	0.4	5	B1	1.00	0.04	0.4	5
B1	0.99	0.04	0.8	5	B1	0.95	0.05	0.8	5	B1	0.95	0.05	0.8	5
B1	0.98	0.03	0.1	10	B1	0.97	0.03	0.1	10	B1	0.97	0.03	0.1	10
B1	1.01	0.03	0.2	10	B1	1.00	0.03	0.2	10	B1	0.96	0.01	0.2	10
B1	0.95	0.03	0.3	10	B1	0.94	0.03	0.3	10	B1	0.96	0.01	0.3	10
B1	0.97	0.03	0.4	10	B1	0.95	0.03	0.4	10	B1	0.95	0.03	0.4	10
B1	0.95	0.03	0.8	10	B1	0.95	0.03	0.8	10	B1	0.95	0.03	0.8	10
B1	1.01	0.02	0.1	20	B1	1.01	0.02	0.1	20	B1	1.01	0.02	0.1	20
B1	0.98	0.02	0.2	20	B1	0.97	0.02	0.2	20	B1	0.96	0.00	0.2	20
B1	0.95	0.02	0.3	20	B1	0.95	0.02	0.3	20	B1	0.96	0.00	0.3	20
B1	0.96	0.02	0.4	20	B1	0.96	0.02	0.4	20	B1	0.96	0.02	0.4	20
B1	0.98	0.02	0.8	20	B1	0.98	0.02	0.8	20	B1	0.98	0.02	0.8	20
B1	0.99	0.01	0.1	40	B1	1.00	0.01	0.1	40	B1	0.96	0.00	0.1	40
B1	0.99	0.01	0.2	40	B1	0.99	0.01	0.2	40	B1	0.96	0.00	0.2	40
B1	1.02	0.01	0.3	40	B1	1.02	0.01	0.3	40	B1	0.96	0.00	0.3	40
B1	0.97	0.01	0.4	40	B1	0.97	0.01	0.4	40	B1	0.96	0.00	0.4	40
B1	0.99	0.02	0.8	40	B1	0.98	0.02	0.8	40	B1	0.96	0.00	0.8	40
B1	0.99	0.01	0.1	80	B1	0.99	0.01	0.1	80	B1	0.96	0.00	0.1	80
B1	0.98	0.01	0.2	80	B1	0.98	0.01	0.2	80	B1	0.96	0.00	0.2	80
B1	0.99	0.01	0.3	80	B1	0.99	0.01	0.3	80	B1	0.96	0.00	0.3	80
B1	0.97	0.01	0.4	80	B1	0.97	0.01	0.4	80	B1	0.96	0.00	0.4	80
B1	0.98	0.01	0.8	80	B1	0.98	0.01	0.8	80	B1	0.97	0.00	0.8	80
B2	2.51	0.35	0.1	5	B2	1.05	0.08	0.1	5	B2	2.17	1.08	0.1	5
B2	1.30	0.17	0.2	5	B2	1.05	0.07	0.2	5	B2	1.78	0.06	0.2	5
B2	1.79	0.12	0.3	5	B2	1.22	0.07	0.3	5	B2	2.11	0.05	0.3	5
B2	1.84	0.10	0.4	5	B2	1.33	0.07	0.4	5	B2	1.77	0.15	0.4	5
B2	1.95	0.07	0.8	5	B2	1.71	0.06	0.8	5	B2	1.96	0.07	0.8	5

B2	2.23	0.28	0.1	10	B2	1.14	0.09	0.1	10	B2	3.28	1.19	0.1	10
B2	2.06	0.14	0.2	10	B2	1.38	0.08	0.2	10	B2	1.86	0.04	0.2	10
B2	1.90	0.09	0.3	10	B2	1.42	0.07	0.3	10	B2	1.81	0.02	0.3	10
B2	1.95	0.07	0.4	10	B2	1.60	0.06	0.4	10	B2	2.00	0.09	0.4	10
B2	1.92	0.05	0.8	10	B2	1.75	0.04	0.8	10	B2	1.88	0.05	0.8	10
B2	1.88	0.21	0.1	20	B2	1.06	0.09	0.1	20	B2	1.54	1.03	0.1	20
B2	2.00	0.11	0.2	20	B2	1.46	0.08	0.2	20	B2	1.80	0.03	0.2	20
B2	2.05	0.07	0.3	20	B2	1.69	0.06	0.3	20	B2	1.97	0.01	0.3	20
B2	1.86	0.06	0.4	20	B2	1.61	0.05	0.4	20	B2	1.82	0.07	0.4	20
B2	1.88	0.04	0.8	20	B2	1.80	0.04	0.8	20	B2	1.86	0.04	0.8	20
B2	2.00	0.18	0.1	40	B2	1.16	0.10	0.1	40	B2	1.97	0.04	0.1	40
B2	2.09	0.09	0.2	40	B2	1.73	0.07	0.2	40	B2	2.00	0.01	0.2	40
B2	2.01	0.06	0.3	40	B2	1.78	0.06	0.3	40	B2	1.95	0.01	0.3	40
B2	2.00	0.05	0.4	40	B2	1.88	0.05	0.4	40	B2	1.95	0.01	0.4	40
B2	1.97	0.03	0.8	40	B2	1.95	0.03	0.8	40	B2	1.95	0.00	0.8	40
B2	2.05	0.16	0.1	80	B2	1.36	0.11	0.1	80	B2	1.81	0.02	0.1	80
B2	1.93	0.08	0.2	80	B2	1.73	0.07	0.2	80	B2	1.92	0.01	0.2	80
B2	2.03	0.06	0.3	80	B2	1.90	0.06	0.3	80	B2	2.00	0.01	0.3	80
B2	1.93	0.04	0.4	80	B2	1.85	0.04	0.4	80	B2	1.87	0.00	0.4	80
B2	1.93	0.02	0.8	80	B2	1.90	0.02	0.8	80	B2	1.89	0.00	0.8	80
Ψ	0.13	0.05	0.1	5	Ψ	0.15	0.06	0.1	5	Ψ	0.15	0.06	0.1	5
Ψ	0.07	0.05	0.2	5	Ψ	0.07	0.05	0.2	5	Ψ	0.04	0.00	0.2	5
Ψ	0.11	0.05	0.3	5	Ψ	0.14	0.05	0.3	5	Ψ	0.04	0.00	0.3	5
Ψ	0.10	0.06	0.4	5	Ψ	0.16	0.06	0.4	5	Ψ	0.16	0.06	0.4	5
Ψ	0.09	0.06	0.8	5	Ψ	0.17	0.06	0.8	5	Ψ	0.17	0.06	0.8	5
Ψ	0.19	0.03	0.1	10	Ψ	0.20	0.03	0.1	10	Ψ	0.20	0.03	0.1	10
Ψ	0.19	0.03	0.2	10	Ψ	0.21	0.03	0.2	10	Ψ	0.04	0.00	0.2	10
Ψ	0.16	0.03	0.3	10	Ψ	0.20	0.03	0.3	10	Ψ	0.04	0.00	0.3	10
Ψ	0.11	0.03	0.4	10	Ψ	0.16	0.03	0.4	10	Ψ	0.16	0.03	0.4	10
Ψ	0.14	0.03	0.8	10	Ψ	0.22	0.04	0.8	10	Ψ	0.22	0.04	0.8	10
Ψ	0.19	0.02	0.1	20	Ψ	0.20	0.02	0.1	20	Ψ	0.20	0.02	0.1	20
Ψ	0.18	0.02	0.2	20	Ψ	0.20	0.02	0.2	20	Ψ	0.04	0.00	0.2	20
Ψ	0.17	0.02	0.3	20	Ψ	0.20	0.02	0.3	20	Ψ	0.04	0.00	0.3	20

Ψ	0.22	0.02	0.4	20	Ψ	0.25	0.02	0.4	20	Ψ	0.25	0.02	0.4	20
Ψ	0.19	0.02	0.8	20	Ψ	0.24	0.03	0.8	20	Ψ	0.24	0.03	0.8	20
Ψ	0.20	0.02	0.1	40	Ψ	0.21	0.02	0.1	40	Ψ	0.04	0.00	0.1	40
Ψ	0.16	0.01	0.2	40	Ψ	0.18	0.01	0.2	40	Ψ	0.04	0.00	0.2	40
Ψ	0.19	0.02	0.3	40	Ψ	0.21	0.02	0.3	40	Ψ	0.04	0.00	0.3	40
Ψ	0.16	0.01	0.4	40	Ψ	0.18	0.01	0.4	40	Ψ	0.04	0.00	0.4	40
Ψ	0.19	0.02	0.8	40	Ψ	0.21	0.02	0.8	40	Ψ	0.03	0.00	0.8	40
Ψ	0.20	0.01	0.1	80	Ψ	0.20	0.01	0.1	80	Ψ	0.04	0.00	0.1	80
Ψ	0.19	0.01	0.2	80	Ψ	0.20	0.01	0.2	80	Ψ	0.04	0.00	0.2	80
Ψ	0.21	0.01	0.3	80	Ψ	0.22	0.01	0.3	80	Ψ	0.04	0.00	0.3	80
Ψ	0.20	0.01	0.4	80	Ψ	0.21	0.01	0.4	80	Ψ	0.03	0.00	0.4	80
Ψ	0.20	0.01	0.8	80	Ψ	0.21	0.01	0.8	80	Ψ	0.03	0.00	0.8	80

Table B.2. Logit, Ψ=0.5

Ψ=0.5	Model A					Model B					Model C			
	Estimate	SE	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj
B0	0.91	0.04	0.1	5	B0	0.90	0.04	0.1	5	B0	0.88	0.04	0.1	5
B0	0.88	0.04	0.2	5	B0	0.89	0.04	0.2	5	B0	0.90	0.01	0.2	5
B0	0.90	0.04	0.3	5	B0	0.89	0.04	0.3	5	B0	0.87	0.01	0.3	5
B0	1.00	0.04	0.4	5	B0	1.00	0.04	0.4	5	B0	0.99	0.04	0.4	5
B0	0.88	0.04	0.8	5	B0	0.86	0.04	0.8	5	B0	0.86	0.04	0.8	5
B0	1.00	0.03	0.1	10	B0	1.00	0.03	0.1	10	B0	1.00	0.03	0.1	10
B0	0.92	0.03	0.2	10	B0	0.92	0.03	0.2	10	B0	0.89	0.01	0.2	10
B0	0.97	0.03	0.3	10	B0	0.98	0.03	0.3	10	B0	0.92	0.01	0.3	10
B0	0.95	0.03	0.4	10	B0	0.94	0.04	0.4	10	B0	0.94	0.04	0.4	10
B0	0.94	0.04	0.8	10	B0	0.93	0.04	0.8	10	B0	0.93	0.04	0.8	10
B0	1.00	0.03	0.1	20	B0	1.00	0.03	0.1	20	B0	1.01	0.03	0.1	20
B0	0.94	0.03	0.2	20	B0	0.93	0.03	0.2	20	B0	0.89	0.01	0.2	20
B0	0.96	0.03	0.3	20	B0	0.96	0.03	0.3	20	B0	0.90	0.01	0.3	20
B0	0.89	0.03	0.4	20	B0	0.89	0.03	0.4	20	B0	0.89	0.03	0.4	20
B0	0.97	0.03	0.8	20	B0	0.97	0.03	0.8	20	B0	0.97	0.03	0.8	20
B0	1.01	0.03	0.1	40	B0	1.01	0.03	0.1	40	B0	0.93	0.00	0.1	40

B0	1.00	0.02	0.2	40	B0	1.01	0.03	0.2	40	B0	0.94	0.00	0.2	40
B0	0.99	0.03	0.3	40	B0	0.98	0.03	0.3	40	B0	0.91	0.00	0.3	40
B0	0.97	0.03	0.4	40	B0	0.97	0.03	0.4	40	B0	0.90	0.00	0.4	40
B0	0.98	0.03	0.8	40	B0	0.97	0.03	0.8	40	B0	0.93	0.00	0.8	40
B0	0.97	0.02	0.1	80	B0	0.97	0.02	0.1	80	B0	0.90	0.00	0.1	80
B0	0.99	0.02	0.2	80	B0	0.98	0.02	0.2	80	B0	0.88	0.00	0.2	80
B0	0.97	0.02	0.3	80	B0	0.97	0.02	0.3	80	B0	0.90	0.00	0.3	80
B0	0.96	0.02	0.4	80	B0	0.96	0.02	0.4	80	B0	0.88	0.00	0.4	80
B0	0.96	0.02	0.8	80	B0	0.96	0.02	0.8	80	B0	0.90	0.00	0.8	80
B1	0.86	0.04	0.1	5	B1	0.84	0.04	0.1	5	B1	0.84	0.04	0.1	5
B1	0.93	0.04	0.2	5	B1	0.92	0.04	0.2	5	B1	0.92	0.01	0.2	5
B1	0.91	0.04	0.3	5	B1	0.92	0.04	0.3	5	B1	0.90	0.01	0.3	5
B1	0.97	0.04	0.4	5	B1	0.97	0.04	0.4	5	B1	0.97	0.04	0.4	5
B1	0.94	0.04	0.8	5	B1	0.93	0.04	0.8	5	B1	0.93	0.04	0.8	5
B1	0.94	0.03	0.1	10	B1	0.94	0.03	0.1	10	B1	0.94	0.03	0.1	10
B1	0.95	0.03	0.2	10	B1	0.95	0.03	0.2	10	B1	0.91	0.01	0.2	10
B1	0.97	0.03	0.3	10	B1	0.96	0.03	0.3	10	B1	0.91	0.01	0.3	10
B1	0.97	0.03	0.4	10	B1	0.98	0.03	0.4	10	B1	0.98	0.03	0.4	10
B1	1.00	0.03	0.8	10	B1	0.99	0.03	0.8	10	B1	0.99	0.03	0.8	10
B1	1.00	0.02	0.1	20	B1	0.99	0.02	0.1	20	B1	0.99	0.02	0.1	20
B1	0.98	0.02	0.2	20	B1	0.98	0.02	0.2	20	B1	0.90	0.01	0.2	20
B1	0.96	0.02	0.3	20	B1	0.96	0.02	0.3	20	B1	0.91	0.01	0.3	20
B1	0.99	0.02	0.4	20	B1	0.98	0.02	0.4	20	B1	0.98	0.02	0.4	20
B1	0.97	0.02	0.8	20	B1	0.97	0.02	0.8	20	B1	0.97	0.02	0.8	20
B1	0.99	0.01	0.1	40	B1	0.98	0.01	0.1	40	B1	0.91	0.00	0.1	40
B1	0.98	0.01	0.2	40	B1	0.97	0.01	0.2	40	B1	0.92	0.00	0.2	40
B1	0.99	0.01	0.3	40	B1	0.99	0.01	0.3	40	B1	0.91	0.00	0.3	40
B1	0.99	0.01	0.4	40	B1	0.99	0.01	0.4	40	B1	0.91	0.00	0.4	40
B1	0.99	0.02	0.8	40	B1	0.99	0.02	0.8	40	B1	0.92	0.00	0.8	40
B1	1.00	0.01	0.1	80	B1	1.00	0.01	0.1	80	B1	0.91	0.00	0.1	80
B1	0.99	0.01	0.2	80	B1	0.99	0.01	0.2	80	B1	0.91	0.00	0.2	80
B1	0.99	0.01	0.3	80	B1	0.99	0.01	0.3	80	B1	0.92	0.00	0.3	80
B1	0.99	0.01	0.4	80	B1	0.99	0.01	0.4	80	B1	0.91	0.00	0.4	80

B1	0.99	0.01	0.8	80	B1	0.99	0.01	0.8	80	B1	0.92	0.00	0.8	80
B2	2.28	0.38	0.1	5	B2	0.97	0.09	0.1	5	B2	7.58	4.79	0.1	5
B2	1.76	0.20	0.2	5	B2	1.11	0.08	0.2	5	B2	1.33	0.12	0.2	5
B2	1.78	0.13	0.3	5	B2	1.14	0.08	0.3	5	B2	1.72	0.07	0.3	5
B2	1.95	0.11	0.4	5	B2	1.39	0.07	0.4	5	B2	1.91	0.16	0.4	5
B2	1.88	0.07	0.8	5	B2	1.62	0.06	0.8	5	B2	1.85	0.08	0.8	5
B2	1.63	0.32	0.1	10	B2	0.96	0.09	0.1	10	B2	1.16	0.78	0.1	10
B2	1.68	0.16	0.2	10	B2	1.12	0.09	0.2	10	B2	1.75	0.07	0.2	10
B2	1.87	0.11	0.3	10	B2	1.44	0.08	0.3	10	B2	1.89	0.04	0.3	10
B2	1.83	0.09	0.4	10	B2	1.41	0.07	0.4	10	B2	1.66	0.11	0.4	10
B2	2.03	0.05	0.8	10	B2	1.89	0.05	0.8	10	B2	2.02	0.06	0.8	10
B2	1.63	0.27	0.1	20	B2	1.18	0.11	0.1	20	B2	1.86	0.52	0.1	20
B2	1.91	0.15	0.2	20	B2	1.41	0.10	0.2	20	B2	1.76	0.04	0.2	20
B2	1.90	0.09	0.3	20	B2	1.50	0.08	0.3	20	B2	1.81	0.02	0.3	20
B2	1.94	0.07	0.4	20	B2	1.75	0.06	0.4	20	B2	1.99	0.08	0.4	20
B2	1.91	0.04	0.8	20	B2	1.81	0.04	0.8	20	B2	1.88	0.04	0.8	20
B2	1.83	0.25	0.1	40	B2	1.48	0.13	0.1	40	B2	2.09	0.06	0.1	40
B2	2.27	0.12	0.2	40	B2	1.82	0.10	0.2	40	B2	2.16	0.02	0.2	40
B2	2.14	0.09	0.3	40	B2	1.89	0.08	0.3	40	B2	2.00	0.01	0.3	40
B2	1.86	0.07	0.4	40	B2	1.73	0.06	0.4	40	B2	1.77	0.01	0.4	40
B2	1.98	0.04	0.8	40	B2	1.95	0.04	0.8	40	B2	1.87	0.01	0.8	40
B2	2.04	0.24	0.1	80	B2	1.54	0.16	0.1	80	B2	1.99	0.04	0.1	80
B2	2.24	0.12	0.2	80	B2	2.00	0.11	0.2	80	B2	2.12	0.01	0.2	80
B2	1.91	0.08	0.3	80	B2	1.77	0.07	0.3	80	B2	1.77	0.01	0.3	80
B2	1.91	0.06	0.4	80	B2	1.81	0.06	0.4	80	B2	1.76	0.01	0.4	80
B2	1.93	0.03	0.8	80	B2	1.91	0.03	0.8	80	B2	1.83	0.00	0.8	80
Ψ	0.37	0.06	0.1	5	Ψ	0.38	0.07	0.1	5	Ψ	0.38	0.07	0.1	5
Ψ	0.37	0.07	0.2	5	Ψ	0.38	0.07	0.2	5	Ψ	0.09	0.00	0.2	5
Ψ	0.37	0.07	0.3	5	Ψ	0.42	0.07	0.3	5	Ψ	0.10	0.00	0.3	5
Ψ	0.36	0.07	0.4	5	Ψ	0.42	0.07	0.4	5	Ψ	0.42	0.07	0.4	5
Ψ	0.34	0.07	0.8	5	Ψ	0.46	0.08	0.8	5	Ψ	0.46	0.08	0.8	5
Ψ	0.33	0.04	0.1	10	Ψ	0.34	0.04	0.1	10	Ψ	0.34	0.04	0.1	10
Ψ	0.45	0.05	0.2	10	Ψ	0.46	0.05	0.2	10	Ψ	0.08	0.00	0.2	10

Ψ	0.41	0.05	0.3	10	Ψ	0.43	0.05	0.3	10	Ψ	0.08	0.00	0.3	10
Ψ	0.49	0.05	0.4	10	Ψ	0.54	0.05	0.4	10	Ψ	0.54	0.05	0.4	10
Ψ	0.46	0.05	0.8	10	Ψ	0.53	0.06	0.8	10	Ψ	0.53	0.06	0.8	10
Ψ	0.45	0.03	0.1	20	Ψ	0.45	0.03	0.1	20	Ψ	0.45	0.03	0.1	20
Ψ	0.54	0.04	0.2	20	Ψ	0.55	0.04	0.2	20	Ψ	0.09	0.00	0.2	20
Ψ	0.43	0.03	0.3	20	Ψ	0.46	0.04	0.3	20	Ψ	0.08	0.00	0.3	20
Ψ	0.38	0.03	0.4	20	Ψ	0.41	0.03	0.4	20	Ψ	0.41	0.03	0.4	20
Ψ	0.44	0.04	0.8	20	Ψ	0.48	0.04	0.8	20	Ψ	0.48	0.04	0.8	20
Ψ	0.49	0.03	0.1	40	Ψ	0.48	0.03	0.1	40	Ψ	0.08	0.00	0.1	40
Ψ	0.45	0.03	0.2	40	Ψ	0.47	0.03	0.2	40	Ψ	0.08	0.00	0.2	40
Ψ	0.49	0.03	0.3	40	Ψ	0.52	0.03	0.3	40	Ψ	0.09	0.00	0.3	40
Ψ	0.46	0.03	0.4	40	Ψ	0.47	0.03	0.4	40	Ψ	0.08	0.00	0.4	40
Ψ	0.46	0.03	0.8	40	Ψ	0.48	0.03	0.8	40	Ψ	0.07	0.00	0.8	40
Ψ	0.50	0.03	0.1	80	Ψ	0.51	0.03	0.1	80	Ψ	0.09	0.00	0.1	80
Ψ	0.51	0.03	0.2	80	Ψ	0.52	0.03	0.2	80	Ψ	0.08	0.00	0.2	80
Ψ	0.44	0.02	0.3	80	Ψ	0.46	0.02	0.3	80	Ψ	0.08	0.00	0.3	80
Ψ	0.45	0.02	0.4	80	Ψ	0.47	0.03	0.4	80	Ψ	0.08	0.00	0.4	80
Ψ	0.47	0.03	0.8	80	Ψ	0.48	0.03	0.8	80	Ψ	0.07	0.00	0.8	80

Table B.3. Logit, Ψ=1.0

Ψ=1.0	Model A					Model B					Model C			
	Estimate	SE	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj
B0	0.81	0.04	0.1	5	B0	0.81	0.04	0.1	5	B0	0.80	0.04	0.1	5
B0	0.91	0.05	0.2	5	B0	0.90	0.05	0.2	5	B0	0.80	0.01	0.2	5
B0	0.88	0.04	0.3	5	B0	0.88	0.04	0.3	5	B0	0.87	0.01	0.3	5
B0	0.85	0.04	0.4	5	B0	0.83	0.04	0.4	5	B0	0.82	0.04	0.4	5
B0	0.87	0.05	0.8	5	B0	0.86	0.05	0.8	5	B0	0.86	0.05	0.8	5
B0	0.92	0.04	0.1	10	B0	0.92	0.04	0.1	10	B0	0.91	0.04	0.1	10
B0	0.90	0.04	0.2	10	B0	0.91	0.04	0.2	10	B0	0.86	0.01	0.2	10
B0	0.89	0.04	0.3	10	B0	0.88	0.04	0.3	10	B0	0.82	0.01	0.3	10
B0	0.89	0.04	0.4	10	B0	0.89	0.04	0.4	10	B0	0.89	0.04	0.4	10
B0	0.99	0.04	0.8	10	B0	0.98	0.04	0.8	10	B0	0.98	0.04	0.8	10

B0	0.93	0.04	0.1	20	B0	0.93	0.04	0.1	20	B0	0.93	0.04	0.1	20
B0	0.94	0.04	0.2	20	B0	0.94	0.04	0.2	20	B0	0.84	0.01	0.2	20
B0	1.00	0.04	0.3	20	B0	0.99	0.04	0.3	20	B0	0.85	0.01	0.3	20
B0	0.99	0.04	0.4	20	B0	0.99	0.04	0.4	20	B0	0.99	0.04	0.4	20
B0	0.97	0.04	0.8	20	B0	0.97	0.04	0.8	20	B0	0.97	0.04	0.8	20
B0	1.05	0.03	0.1	40	B0	1.04	0.03	0.1	40	B0	0.88	0.00	0.1	40
B0	0.98	0.03	0.2	40	B0	0.98	0.03	0.2	40	B0	0.85	0.00	0.2	40
B0	1.00	0.03	0.3	40	B0	1.00	0.04	0.3	40	B0	0.85	0.00	0.3	40
B0	1.00	0.03	0.4	40	B0	0.99	0.03	0.4	40	B0	0.87	0.00	0.4	40
B0	1.00	0.03	0.8	40	B0	1.01	0.03	0.8	40	B0	0.90	0.00	0.8	40
B0	0.96	0.03	0.1	80	B0	0.96	0.03	0.1	80	B0	0.84	0.00	0.1	80
B0	0.99	0.03	0.2	80	B0	0.99	0.03	0.2	80	B0	0.84	0.00	0.2	80
B0	0.96	0.03	0.3	80	B0	0.96	0.03	0.3	80	B0	0.82	0.00	0.3	80
B0	0.98	0.03	0.4	80	B0	0.98	0.03	0.4	80	B0	0.81	0.00	0.4	80
B0	0.96	0.03	0.8	80	B0	0.96	0.03	0.8	80	B0	0.83	0.00	0.8	80
B1	0.90	0.04	0.1	5	B1	0.93	0.04	0.1	5	B1	0.93	0.04	0.1	5
B1	0.92	0.04	0.2	5	B1	0.92	0.04	0.2	5	B1	0.84	0.02	0.2	5
B1	0.86	0.04	0.3	5	B1	0.87	0.04	0.3	5	B1	0.83	0.02	0.3	5
B1	0.86	0.04	0.4	5	B1	0.85	0.04	0.4	5	B1	0.85	0.04	0.4	5
B1	0.91	0.04	0.8	5	B1	0.90	0.05	0.8	5	B1	0.90	0.05	0.8	5
B1	0.85	0.03	0.1	10	B1	0.85	0.03	0.1	10	B1	0.85	0.03	0.1	10
B1	0.92	0.03	0.2	10	B1	0.91	0.03	0.2	10	B1	0.85	0.01	0.2	10
B1	0.94	0.03	0.3	10	B1	0.94	0.03	0.3	10	B1	0.84	0.01	0.3	10
B1	0.92	0.03	0.4	10	B1	0.92	0.03	0.4	10	B1	0.92	0.03	0.4	10
B1	0.91	0.03	0.8	10	B1	0.91	0.03	0.8	10	B1	0.91	0.03	0.8	10
B1	0.98	0.02	0.1	20	B1	0.98	0.02	0.1	20	B1	0.98	0.02	0.1	20
B1	0.94	0.02	0.2	20	B1	0.94	0.02	0.2	20	B1	0.83	0.01	0.2	20
B1	0.97	0.02	0.3	20	B1	0.97	0.02	0.3	20	B1	0.84	0.01	0.3	20
B1	0.96	0.02	0.4	20	B1	0.97	0.02	0.4	20	B1	0.97	0.02	0.4	20
B1	0.99	0.02	0.8	20	B1	0.98	0.02	0.8	20	B1	0.98	0.02	0.8	20
B1	1.01	0.01	0.1	40	B1	1.01	0.01	0.1	40	B1	0.85	0.01	0.1	40
B1	0.98	0.01	0.2	40	B1	0.98	0.01	0.2	40	B1	0.84	0.01	0.2	40
B1	0.98	0.01	0.3	40	B1	0.98	0.01	0.3	40	B1	0.84	0.01	0.3	40

B1	0.96	0.01	0.4	40	B1	0.95	0.01	0.4	40	B1	0.85	0.01	0.4	40
B1	0.97	0.02	0.8	40	B1	0.97	0.02	0.8	40	B1	0.85	0.01	0.8	40
B1	0.96	0.01	0.1	80	B1	0.97	0.01	0.1	80	B1	0.84	0.00	0.1	80
B1	0.98	0.01	0.2	80	B1	0.98	0.01	0.2	80	B1	0.84	0.00	0.2	80
B1	0.99	0.01	0.3	80	B1	0.99	0.01	0.3	80	B1	0.85	0.00	0.3	80
B1	1.00	0.01	0.4	80	B1	1.00	0.01	0.4	80	B1	0.83	0.00	0.4	80
B1	0.99	0.01	0.8	80	B1	0.99	0.01	0.8	80	B1	0.85	0.00	0.8	80
B2	2.41	0.40	0.1	5	B2	0.80	0.09	0.1	5	B2	-1.14	1.56	0.1	5
B2	1.83	0.23	0.2	5	B2	1.10	0.09	0.2	5	B2	1.70	0.16	0.2	5
B2	1.68	0.14	0.3	5	B2	1.05	0.08	0.3	5	B2	1.54	0.08	0.3	5
B2	1.59	0.11	0.4	5	B2	1.18	0.08	0.4	5	B2	1.63	0.18	0.4	5
B2	1.76	0.07	0.8	5	B2	1.54	0.06	0.8	5	B2	1.74	0.08	0.8	5
B2	1.61	0.39	0.1	10	B2	0.95	0.11	0.1	10	B2	1.59	0.80	0.1	10
B2	1.69	0.18	0.2	10	B2	1.31	0.10	0.2	10	B2	1.90	0.08	0.2	10
B2	1.90	0.13	0.3	10	B2	1.29	0.09	0.3	10	B2	1.68	0.05	0.3	10
B2	1.78	0.09	0.4	10	B2	1.46	0.07	0.4	10	B2	1.75	0.11	0.4	10
B2	1.82	0.06	0.8	10	B2	1.69	0.06	0.8	10	B2	1.82	0.06	0.8	10
B2	1.65	0.34	0.1	20	B2	1.04	0.14	0.1	20	B2	1.32	0.76	0.1	20
B2	1.64	0.19	0.2	20	B2	1.24	0.13	0.2	20	B2	1.43	0.06	0.2	20
B2	1.85	0.12	0.3	20	B2	1.50	0.10	0.3	20	B2	1.64	0.03	0.3	20
B2	1.99	0.09	0.4	20	B2	1.67	0.08	0.4	20	B2	1.90	0.11	0.4	20
B2	1.99	0.05	0.8	20	B2	1.92	0.05	0.8	20	B2	2.00	0.06	0.8	20
B2	2.13	0.34	0.1	40	B2	1.33	0.18	0.1	40	B2	1.65	0.09	0.1	40
B2	2.01	0.17	0.2	40	B2	1.67	0.13	0.2	40	B2	1.67	0.03	0.2	40
B2	1.99	0.12	0.3	40	B2	1.79	0.11	0.3	40	B2	1.70	0.02	0.3	40
B2	2.05	0.09	0.4	40	B2	1.94	0.08	0.4	40	B2	1.79	0.01	0.4	40
B2	1.95	0.05	0.8	40	B2	1.93	0.05	0.8	40	B2	1.76	0.01	0.8	40
B2	2.04	0.32	0.1	80	B2	1.40	0.21	0.1	80	B2	1.81	0.05	0.1	80
B2	2.19	0.17	0.2	80	B2	1.87	0.15	0.2	80	B2	1.93	0.02	0.2	80
B2	2.18	0.11	0.3	80	B2	2.02	0.10	0.3	80	B2	1.90	0.01	0.3	80
B2	2.05	0.09	0.4	80	B2	1.98	0.08	0.4	80	B2	1.70	0.01	0.4	80
B2	1.94	0.04	0.8	80	B2	1.92	0.04	0.8	80	B2	1.73	0.01	0.8	80
Ψ	0.57	0.08	0.1	5	Ψ	0.58	0.08	0.1	5	Ψ	0.58	0.08	0.1	5

Ψ	0.83	0.09	0.2	5	Ψ	0.84	0.09	0.2	5	Ψ	0.16	0.00	0.2	5
Ψ	0.60	0.08	0.3	5	Ψ	0.63	0.08	0.3	5	Ψ	0.15	0.00	0.3	5
Ψ	0.67	0.08	0.4	5	Ψ	0.71	0.08	0.4	5	Ψ	0.71	0.08	0.4	5
Ψ	0.64	0.09	0.8	5	Ψ	0.70	0.09	0.8	5	Ψ	0.70	0.09	0.8	5
Ψ	0.78	0.06	0.1	10	Ψ	0.79	0.06	0.1	10	Ψ	0.79	0.06	0.1	10
Ψ	0.76	0.06	0.2	10	Ψ	0.77	0.06	0.2	10	Ψ	0.14	0.00	0.2	10
Ψ	0.79	0.06	0.3	10	Ψ	0.84	0.07	0.3	10	Ψ	0.15	0.00	0.3	10
Ψ	0.69	0.06	0.4	10	Ψ	0.73	0.06	0.4	10	Ψ	0.73	0.06	0.4	10
Ψ	0.81	0.07	0.8	10	Ψ	0.86	0.07	0.8	10	Ψ	0.86	0.07	0.8	10
Ψ	0.91	0.06	0.1	20	Ψ	0.92	0.06	0.1	20	Ψ	0.92	0.06	0.1	20
Ψ	1.03	0.06	0.2	20	Ψ	1.04	0.06	0.2	20	Ψ	0.16	0.00	0.2	20
Ψ	0.96	0.06	0.3	20	Ψ	0.98	0.06	0.3	20	Ψ	0.15	0.00	0.3	20
Ψ	0.93	0.06	0.4	20	Ψ	0.99	0.06	0.4	20	Ψ	0.99	0.06	0.4	20
Ψ	0.95	0.06	0.8	20	Ψ	0.98	0.06	0.8	20	Ψ	0.98	0.06	0.8	20
Ψ	0.94	0.05	0.1	40	Ψ	0.95	0.05	0.1	40	Ψ	0.14	0.00	0.1	40
Ψ	0.91	0.05	0.2	40	Ψ	0.92	0.05	0.2	40	Ψ	0.15	0.00	0.2	40
Ψ	1.03	0.06	0.3	40	Ψ	1.04	0.06	0.3	40	Ψ	0.15	0.00	0.3	40
Ψ	0.91	0.05	0.4	40	Ψ	0.92	0.05	0.4	40	Ψ	0.14	0.00	0.4	40
Ψ	0.86	0.05	0.8	40	Ψ	0.87	0.05	0.8	40	Ψ	0.13	0.00	0.8	40
Ψ	0.93	0.05	0.1	80	Ψ	0.93	0.05	0.1	80	Ψ	0.15	0.00	0.1	80
Ψ	1.02	0.05	0.2	80	Ψ	1.04	0.05	0.2	80	Ψ	0.15	0.00	0.2	80
Ψ	0.96	0.05	0.3	80	Ψ	0.98	0.05	0.3	80	Ψ	0.14	0.00	0.3	80
Ψ	1.09	0.05	0.4	80	Ψ	1.10	0.06	0.4	80	Ψ	0.16	0.00	0.4	80
Ψ	0.94	0.05	0.8	80	Ψ	0.96	0.05	0.8	80	Ψ	0.13	0.00	0.8	80

2. Probit Simulations

Table B.4. Probit, Ψ=0.2

Ψ=0.2	Model A					Model B					Model C		
	Estimate	SE	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj
B0	0.99	0.03	0.1	5	B0	0.99	0.03	0.1	5	B0	0.98	0.03	0.1	5
B0	0.97	0.03	0.2	5	B0	0.97	0.03	0.2	5	B0	0.97	0.03	0.2	5
B0	0.96	0.03	0.3	5	B0	0.97	0.03	0.3	5	B0	0.99	0.04	0.3	5
B0	0.97	0.03	0.4	5	B0	0.98	0.04	0.4	5	B0	0.96	0.04	0.4	5
B0	1.05	0.04	0.8	5	B0	1.09	0.05	0.8	5	B0	1.10	0.05	0.8	5
B0	1.01	0.02	0.1	10	B0	1.01	0.02	0.1	10	B0	0.98	0.03	0.1	10
B0	0.97	0.02	0.2	10	B0	0.97	0.02	0.2	10	B0	0.97	0.02	0.2	10
B0	1.01	0.03	0.3	10	B0	1.03	0.03	0.3	10	B0	1.04	0.03	0.3	10
B0	1.00	0.03	0.4	10	B0	1.00	0.03	0.4	10	B0	1.00	0.03	0.4	10
B0	1.57	0.13	0.8	10	B0	0.98	0.03	0.8	10	B0	0.97	0.03	0.8	10
B0	1.03	0.02	0.1	20	B0	1.02	0.02	0.1	20	B0	1.03	0.02	0.1	20
B0	0.97	0.02	0.2	20	B0	0.97	0.02	0.2	20	B0	0.98	0.02	0.2	20
B0	1.00	0.02	0.3	20	B0	0.99	0.02	0.3	20	B0	0.99	0.02	0.3	20
B0	1.01	0.02	0.4	20	B0	1.01	0.02	0.4	20	B0	1.01	0.02	0.4	20
B0	1.57	0.09	0.8	20	B0	0.99	0.02	0.8	20	B0	0.99	0.02	0.8	20
B0	1.01	0.02	0.1	40	B0	1.01	0.02	0.1	40	B0	1.00	0.02	0.1	40
B0	0.97	0.02	0.2	40	B0	0.97	0.02	0.2	40	B0	0.97	0.02	0.2	40
B0	1.00	0.02	0.3	40	B0	1.00	0.02	0.3	40	B0	1.00	0.02	0.3	40
B0	1.04	0.02	0.4	40	B0	1.04	0.02	0.4	40	B0	1.04	0.02	0.4	40
B0	1.01	0.02	0.8	40	B0	1.01	0.02	0.8	40	B0	1.01	0.02	0.8	40
B0	0.99	0.02	0.1	80	B0	0.99	0.02	0.1	80	B0	0.99	0.02	0.1	80
B0	1.04	0.02	0.2	80	B0	1.04	0.02	0.2	80	B0	1.04	0.02	0.2	80
B0	0.98	0.02	0.3	80	B0	0.98	0.02	0.3	80	B0	0.98	0.02	0.3	80
B0	0.99	0.02	0.4	80	B0	1.00	0.02	0.4	80	B0	1.00	0.02	0.4	80
B0			0.8	80	B0			0.8	80	B0			0.8	80
B1	1.00	0.03	0.1	5	B1	0.99	0.04	0.1	5	B1	0.99	0.04	0.1	5
B1	0.99	0.03	0.2	5	B1	0.99	0.04	0.2	5	B1	0.99	0.04	0.2	5
B1	0.94	0.03	0.3	5	B1	0.95	0.04	0.3	5	B1	0.95	0.04	0.3	5

B1	1.00	0.04	0.4	5	B1	0.99	0.04	0.4	5	B1	0.99	0.04	0.4	5
B1	1.02	0.04	0.8	5	B1	1.01	0.04	0.8	5	B1	1.01	0.04	0.8	5
B1	0.98	0.02	0.1	10	B1	0.98	0.02	0.1	10	B1	0.98	0.02	0.1	10
B1	0.96	0.02	0.2	10	B1	0.96	0.02	0.2	10	B1	0.96	0.02	0.2	10
B1	1.01	0.02	0.3	10	B1	1.01	0.02	0.3	10	B1	1.01	0.02	0.3	10
B1	1.05	0.03	0.4	10	B1	1.05	0.03	0.4	10	B1	1.05	0.03	0.4	10
B1	1.59	0.13	0.8	10	B1	1.02	0.03	0.8	10	B1	1.02	0.03	0.8	10
B1	0.99	0.02	0.1	20	B1	0.99	0.02	0.1	20	B1	0.99	0.02	0.1	20
B1	1.00	0.02	0.2	20	B1	1.00	0.02	0.2	20	B1	1.00	0.02	0.2	20
B1	1.02	0.02	0.3	20	B1	1.02	0.02	0.3	20	B1	1.02	0.02	0.3	20
B1	0.99	0.02	0.4	20	B1	0.99	0.02	0.4	20	B1	0.99	0.02	0.4	20
B1	1.53	0.09	0.8	20	B1	1.00	0.02	0.8	20	B1	1.00	0.02	0.8	20
B1	0.98	0.01	0.1	40	B1	0.98	0.01	0.1	40	B1	0.98	0.01	0.1	40
B1	0.98	0.01	0.2	40	B1	0.98	0.01	0.2	40	B1	0.98	0.01	0.2	40
B1	1.00	0.01	0.3	40	B1	1.00	0.01	0.3	40	B1	1.00	0.01	0.3	40
B1	1.02	0.01	0.4	40	B1	1.02	0.01	0.4	40	B1	1.02	0.01	0.4	40
B1	1.01	0.01	0.8	40	B1	1.01	0.01	0.8	40	B1	1.01	0.01	0.8	40
B1	1.01	0.01	0.1	80	B1	1.01	0.01	0.1	80	B1	1.01	0.01	0.1	80
B1	1.00	0.01	0.2	80	B1	1.00	0.01	0.2	80	B1	1.00	0.01	0.2	80
B1	1.00	0.01	0.3	80	B1	1.00	0.01	0.3	80	B1	1.00	0.01	0.3	80
B1	1.01	0.01	0.4	80	B1	1.01	0.01	0.4	80	B1	1.01	0.01	0.4	80
B1			0.8	80	B1			0.8	80	B1			0.8	80
B2	1.71	0.27	0.1	5	B2	1.09	0.06	0.1	5	B2	2.34	0.79	0.1	5
B2	1.88	0.14	0.2	5	B2	1.15	0.06	0.2	5	B2	1.73	0.26	0.2	5
B2	2.07	0.10	0.3	5	B2	1.30	0.06	0.3	5	B2	2.18	0.21	0.3	5
B2	1.99	0.09	0.4	5	B2	1.47	0.06	0.4	5	B2	2.12	0.13	0.4	5
B2	2.10	0.07	0.8	5	B2	1.87	0.07	0.8	5	B2	2.16	0.09	0.8	5
B2	2.09	0.22	0.1	10	B2	1.13	0.07	0.1	10	B2	3.06	0.93	0.1	10
B2	1.90	0.11	0.2	10	B2	1.28	0.06	0.2	10	B2	1.99	0.19	0.2	10
B2	1.98	0.08	0.3	10	B2	1.55	0.06	0.3	10	B2	2.09	0.11	0.3	10
B2	1.96	0.07	0.4	10	B2	1.63	0.05	0.4	10	B2	2.00	0.08	0.4	10
B2	3.31	0.24	0.8	10	B2	1.92	0.05	0.8	10	B2	2.07	0.06	0.8	10
B2	2.08	0.19	0.1	20	B2	1.10	0.08	0.1	20	B2	2.20	0.91	0.1	20

B2	1.86	0.10	0.2	20	B2	1.36	0.07	0.2	20	B2	1.82	0.15	0.2	20
B2	2.08	0.07	0.3	20	B2	1.73	0.06	0.3	20	B2	2.12	0.09	0.3	20
B2	1.95	0.05	0.4	20	B2	1.70	0.05	0.4	20	B2	1.92	0.06	0.4	20
B2	3.07	0.17	0.8	20	B2	1.86	0.04	0.8	20	B2	1.93	0.04	0.8	20
B2	2.12	0.17	0.1	40	B2	1.29	0.09	0.1	40	B2	2.04	0.31	0.1	40
B2	2.05	0.08	0.2	40	B2	1.67	0.06	0.2	40	B2	2.07	0.10	0.2	40
B2	2.03	0.06	0.3	40	B2	1.84	0.05	0.3	40	B2	2.06	0.07	0.3	40
B2	2.02	0.05	0.4	40	B2	1.89	0.04	0.4	40	B2	2.02	0.05	0.4	40
B2	2.03	0.03	0.8	40	B2	2.00	0.03	0.8	40	B2	2.05	0.03	0.8	40
B2	1.96	0.15	0.1	80	B2	1.37	0.11	0.1	80	B2	1.83	0.24	0.1	80
B2	2.04	0.08	0.2	80	B2	1.79	0.07	0.2	80	B2	2.03	0.09	0.2	80
B2	2.10	0.05	0.3	80	B2	1.96	0.05	0.3	80	B2	2.10	0.06	0.3	80
B2	1.96	0.04	0.4	80	B2	1.88	0.04	0.4	80	B2	1.95	0.04	0.4	80
B2			0.8	80	B2			0.8	80	B2			0.8	80
Ψ	0.16	0.04	0.1	5	Ψ	0.16	0.04	0.1	5	Ψ	0.16	0.04	0.1	5
Ψ	0.15	0.04	0.2	5	Ψ	0.18	0.04	0.2	5	Ψ	0.18	0.04	0.2	5
Ψ	0.17	0.04	0.3	5	Ψ	0.26	0.04	0.3	5	Ψ	0.26	0.04	0.3	5
Ψ	0.17	0.04	0.4	5	Ψ	0.25	0.05	0.4	5	Ψ	0.25	0.05	0.4	5
Ψ	0.22	0.05	0.8	5	Ψ	0.39	0.06	0.8	5	Ψ	0.39	0.06	0.8	5
Ψ	0.18	0.02	0.1	10	Ψ	0.19	0.02	0.1	10	Ψ	0.19	0.02	0.1	10
Ψ	0.16	0.02	0.2	10	Ψ	0.18	0.02	0.2	10	Ψ	0.18	0.02	0.2	10
Ψ	0.20	0.02	0.3	10	Ψ	0.23	0.03	0.3	10	Ψ	0.23	0.03	0.3	10
Ψ	0.20	0.02	0.4	10	Ψ	0.24	0.03	0.4	10	Ψ	0.24	0.03	0.4	10
Ψ	0.80	0.00	0.8	10	Ψ	0.31	0.04	0.8	10	Ψ	0.31	0.04	0.8	10
Ψ	0.22	0.02	0.1	20	Ψ	0.24	0.02	0.1	20	Ψ	0.24	0.02	0.1	20
Ψ	0.22	0.02	0.2	20	Ψ	0.24	0.02	0.2	20	Ψ	0.24	0.02	0.2	20
Ψ	0.20	0.02	0.3	20	Ψ	0.24	0.02	0.3	20	Ψ	0.24	0.02	0.3	20
Ψ	0.18	0.02	0.4	20	Ψ	0.22	0.02	0.4	20	Ψ	0.22	0.02	0.4	20
Ψ	0.20	0.00	0.8	20	Ψ	0.25	0.02	0.8	20	Ψ	0.25	0.02	0.8	20
Ψ	0.21	0.01	0.1	40	Ψ	0.22	0.01	0.1	40	Ψ	0.22	0.01	0.1	40
Ψ	0.19	0.01	0.2	40	Ψ	0.20	0.01	0.2	40	Ψ	0.20	0.01	0.2	40
Ψ	0.19	0.01	0.3	40	Ψ	0.21	0.01	0.3	40	Ψ	0.21	0.01	0.3	40
Ψ	0.21	0.01	0.4	40	Ψ	0.23	0.01	0.4	40	Ψ	0.23	0.01	0.4	40

Ψ	0.21	0.01	0.8	40	Ψ	0.23	0.02	0.8	40	Ψ	0.23	0.02	0.8	40
Ψ	0.20	0.01	0.1	80	Ψ	0.21	0.01	0.1	80	Ψ	0.21	0.01	0.1	80
Ψ	0.19	0.01	0.2	80	Ψ	0.21	0.01	0.2	80	Ψ	0.21	0.01	0.2	80
Ψ	0.19	0.01	0.3	80	Ψ	0.20	0.01	0.3	80	Ψ	0.20	0.01	0.3	80
Ψ	0.20	0.01	0.4	80	Ψ	0.22	0.01	0.4	80	Ψ	0.22	0.01	0.4	80
Ψ			0.8	80	Ψ			0.8	80	Ψ			0.8	80

Table B.5. Probit, Ψ=0.5

Ψ=0.5	Model A					Model B					Model C			
	Estimate	SE	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj
B0	0.96	0.04	0.1	5	B0	0.95	0.04	0.1	5	B0	0.95	0.04	0.1	5
B0	0.94	0.04	0.2	5	B0	0.95	0.04	0.2	5	B0	0.95	0.04	0.2	5
B0	1.02	0.04	0.3	5	B0	1.01	0.04	0.3	5	B0	1.01	0.04	0.3	5
B0	1.01	0.04	0.4	5	B0	1.02	0.04	0.4	5	B0	1.01	0.04	0.4	5
B0	0.93	0.04	0.8	5	B0	0.93	0.04	0.8	5	B0	0.93	0.05	0.8	5
B0	1.09	0.03	0.1	10	B0	1.09	0.03	0.1	10	B0	1.09	0.03	0.1	10
B0	0.96	0.03	0.2	10	B0	0.96	0.03	0.2	10	B0	0.96	0.03	0.2	10
B0	0.99	0.03	0.3	10	B0	1.00	0.03	0.3	10	B0	0.99	0.03	0.3	10
B0	1.00	0.03	0.4	10	B0	0.98	0.03	0.4	10	B0	1.00	0.03	0.4	10
B0	0.97	0.04	0.8	10	B0	0.96	0.04	0.8	10	B0	0.96	0.04	0.8	10
B0	0.98	0.03	0.1	20	B0	0.98	0.03	0.1	20	B0	0.98	0.03	0.1	20
B0	1.03	0.03	0.2	20	B0	1.02	0.03	0.2	20	B0	1.03	0.03	0.2	20
B0	1.02	0.03	0.3	20	B0	1.02	0.03	0.3	20	B0	1.01	0.03	0.3	20
B0	0.96	0.03	0.4	20	B0	0.96	0.03	0.4	20	B0	0.96	0.03	0.4	20
B0	0.97	0.03	0.8	20	B0	0.97	0.03	0.8	20	B0	0.98	0.03	0.8	20
B0	1.03	0.03	0.1	40	B0	1.03	0.03	0.1	40	B0	1.03	0.03	0.1	40
B0	1.03	0.02	0.2	40	B0	1.03	0.02	0.2	40	B0	1.03	0.02	0.2	40
B0	1.03	0.03	0.3	40	B0	1.03	0.03	0.3	40	B0	1.03	0.03	0.3	40
B0	1.00	0.02	0.4	40	B0	1.00	0.03	0.4	40	B0	1.00	0.03	0.4	40
B0	1.00	0.03	0.8	40	B0	1.00	0.03	0.8	40	B0	1.00	0.03	0.8	40
B0	0.98	0.02	0.1	80	B0	0.98	0.02	0.1	80	B0	0.99	0.02	0.1	80
B0	0.96	0.02	0.2	80	B0	0.96	0.02	0.2	80	B0	0.96	0.02	0.2	80

B0	0.99	0.02	0.3	80	B0	0.99	0.02	0.3	80	B0	0.99	0.02	0.3	80
B0	0.96	0.02	0.4	80	B0	0.96	0.02	0.4	80	B0	0.96	0.02	0.4	80
B0	0.98	0.03	0.8	80	B0	0.98	0.03	0.8	80	B0	0.98	0.03	0.8	80
B1	0.98	0.04	0.1	5	B1	0.98	0.04	0.1	5	B1	0.98	0.04	0.1	5
B1	0.95	0.03	0.2	5	B1	0.96	0.04	0.2	5	B1	0.96	0.04	0.2	5
B1	1.02	0.04	0.3	5	B1	1.03	0.04	0.3	5	B1	1.03	0.04	0.3	5
B1	0.97	0.04	0.4	5	B1	0.98	0.04	0.4	5	B1	0.98	0.04	0.4	5
B1	0.93	0.04	0.8	5	B1	0.94	0.04	0.8	5	B1	0.94	0.04	0.8	5
B1	1.01	0.02	0.1	10	B1	1.01	0.02	0.1	10	B1	1.01	0.02	0.1	10
B1	0.99	0.02	0.2	10	B1	0.99	0.02	0.2	10	B1	0.99	0.02	0.2	10
B1	1.00	0.02	0.3	10	B1	1.00	0.02	0.3	10	B1	1.00	0.02	0.3	10
B1	0.97	0.02	0.4	10	B1	0.97	0.02	0.4	10	B1	0.97	0.02	0.4	10
B1	0.98	0.03	0.8	10	B1	0.98	0.03	0.8	10	B1	0.98	0.03	0.8	10
B1	0.99	0.02	0.1	20	B1	0.99	0.02	0.1	20	B1	0.99	0.02	0.1	20
B1	1.02	0.02	0.2	20	B1	1.02	0.02	0.2	20	B1	1.02	0.02	0.2	20
B1	1.03	0.02	0.3	20	B1	1.03	0.02	0.3	20	B1	1.03	0.02	0.3	20
B1	1.03	0.02	0.4	20	B1	1.03	0.02	0.4	20	B1	1.03	0.02	0.4	20
B1	1.00	0.02	0.8	20	B1	1.00	0.02	0.8	20	B1	1.00	0.02	0.8	20
B1	0.99	0.01	0.1	40	B1	0.99	0.01	0.1	40	B1	0.99	0.01	0.1	40
B1	1.01	0.01	0.2	40	B1	1.01	0.01	0.2	40	B1	1.01	0.01	0.2	40
B1	1.02	0.01	0.3	40	B1	1.02	0.01	0.3	40	B1	1.02	0.01	0.3	40
B1	1.01	0.01	0.4	40	B1	1.01	0.01	0.4	40	B1	1.01	0.01	0.4	40
B1	1.00	0.01	0.8	40	B1	1.00	0.01	0.8	40	B1	1.00	0.01	0.8	40
B1	1.00	0.01	0.1	80	B1	1.00	0.01	0.1	80	B1	1.00	0.01	0.1	80
B1	1.00	0.01	0.2	80	B1	1.00	0.01	0.2	80	B1	1.00	0.01	0.2	80
B1	1.00	0.01	0.3	80	B1	1.00	0.01	0.3	80	B1	1.00	0.01	0.3	80
B1	1.02	0.01	0.4	80	B1	1.02	0.01	0.4	80	B1	1.02	0.01	0.4	80
B1	1.00	0.01	0.8	80	B1	1.00	0.01	0.8	80	B1	1.00	0.01	0.8	80
B2	2.08	0.32	0.1	5	B2	0.99	0.08	0.1	5	B2	1.51	4.00	0.1	5
B2	2.11	0.16	0.2	5	B2	1.05	0.07	0.2	5	B2	1.44	0.37	0.2	5
B2	2.03	0.13	0.3	5	B2	1.28	0.08	0.3	5	B2	1.88	0.25	0.3	5
B2	1.87	0.10	0.4	5	B2	1.33	0.07	0.4	5	B2	1.77	0.15	0.4	5
B2	1.87	0.07	0.8	5	B2	1.64	0.07	0.8	5	B2	1.88	0.08	0.8	5

B2	2.19	0.30	0.1	10	B2	1.02	0.09	0.1	10	B2	1.10	0.72	0.1	10
B2	1.82	0.14	0.2	10	B2	1.25	0.08	0.2	10	B2	1.97	0.29	0.2	10
B2	1.89	0.10	0.3	10	B2	1.44	0.07	0.3	10	B2	2.04	0.17	0.3	10
B2	1.98	0.08	0.4	10	B2	1.54	0.07	0.4	10	B2	1.89	0.10	0.4	10
B2	2.00	0.06	0.8	10	B2	1.86	0.06	0.8	10	B2	1.99	0.06	0.8	10
B2	1.76	0.25	0.1	20	B2	1.18	0.10	0.1	20	B2	1.87	0.48	0.1	20
B2	2.18	0.14	0.2	20	B2	1.43	0.09	0.2	20	B2	1.91	0.19	0.2	20
B2	2.08	0.09	0.3	20	B2	1.65	0.08	0.3	20	B2	2.01	0.12	0.3	20
B2	2.03	0.07	0.4	20	B2	1.83	0.06	0.4	20	B2	2.08	0.08	0.4	20
B2	1.97	0.04	0.8	20	B2	1.87	0.04	0.8	20	B2	1.94	0.05	0.8	20
B2	1.80	0.25	0.1	40	B2	1.29	0.13	0.1	40	B2	1.98	0.44	0.1	40
B2	2.25	0.12	0.2	40	B2	1.83	0.09	0.2	40	B2	2.28	0.14	0.2	40
B2	2.19	0.09	0.3	40	B2	1.95	0.08	0.3	40	B2	2.20	0.10	0.3	40
B2	1.91	0.06	0.4	40	B2	1.78	0.06	0.4	40	B2	1.90	0.07	0.4	40
B2	1.99	0.04	0.8	40	B2	1.96	0.04	0.8	40	B2	1.99	0.04	0.8	40
B2	2.23	0.24	0.1	80	B2	1.50	0.16	0.1	80	B2	2.13	0.37	0.1	80
B2	2.22	0.12	0.2	80	B2	1.99	0.10	0.2	80	B2	2.33	0.14	0.2	80
B2	1.97	0.08	0.3	80	B2	1.81	0.07	0.3	80	B2	1.92	0.08	0.3	80
B2	1.97	0.06	0.4	80	B2	1.88	0.06	0.4	80	B2	1.94	0.06	0.4	80
B2	1.99	0.04	0.8	80	B2	1.97	0.04	0.8	80	B2	1.99	0.04	0.8	80
Ψ	0.48	0.06	0.1	5	Ψ	0.49	0.06	0.1	5	Ψ	0.49	0.06	0.1	5
Ψ	0.38	0.05	0.2	5	Ψ	0.43	0.06	0.2	5	Ψ	0.43	0.06	0.2	5
Ψ	0.58	0.07	0.3	5	Ψ	0.65	0.08	0.3	5	Ψ	0.65	0.08	0.3	5
Ψ	0.55	0.07	0.4	5	Ψ	0.63	0.07	0.4	5	Ψ	0.63	0.07	0.4	5
Ψ	0.46	0.07	0.8	5	Ψ	0.63	0.08	0.8	5	Ψ	0.63	0.08	0.8	5
Ψ	0.46	0.04	0.1	10	Ψ	0.48	0.04	0.1	10	Ψ	0.48	0.04	0.1	10
Ψ	0.48	0.04	0.2	10	Ψ	0.50	0.04	0.2	10	Ψ	0.50	0.04	0.2	10
Ψ	0.49	0.04	0.3	10	Ψ	0.52	0.04	0.3	10	Ψ	0.52	0.04	0.3	10
Ψ	0.50	0.04	0.4	10	Ψ	0.57	0.05	0.4	10	Ψ	0.57	0.05	0.4	10
Ψ	0.50	0.05	0.8	10	Ψ	0.59	0.05	0.8	10	Ψ	0.59	0.05	0.8	10
Ψ	0.48	0.03	0.1	20	Ψ	0.49	0.03	0.1	20	Ψ	0.49	0.03	0.1	20
Ψ	0.55	0.04	0.2	20	Ψ	0.58	0.04	0.2	20	Ψ	0.58	0.04	0.2	20
Ψ	0.51	0.03	0.3	20	Ψ	0.55	0.04	0.3	20	Ψ	0.55	0.04	0.3	20

Ψ	0.48	0.03	0.4	20	Ψ	0.51	0.03	0.4	20	Ψ	0.51	0.03	0.4	20
Ψ	0.45	0.03	0.8	20	Ψ	0.50	0.04	0.8	20	Ψ	0.50	0.04	0.8	20
Ψ	0.53	0.03	0.1	40	Ψ	0.53	0.03	0.1	40	Ψ	0.53	0.03	0.1	40
Ψ	0.47	0.03	0.2	40	Ψ	0.49	0.03	0.2	40	Ψ	0.49	0.03	0.2	40
Ψ	0.54	0.03	0.3	40	Ψ	0.57	0.03	0.3	40	Ψ	0.57	0.03	0.3	40
Ψ	0.49	0.03	0.4	40	Ψ	0.51	0.03	0.4	40	Ψ	0.51	0.03	0.4	40
Ψ	0.49	0.03	0.8	40	Ψ	0.50	0.03	0.8	40	Ψ	0.50	0.03	0.8	40
Ψ	0.52	0.03	0.1	80	Ψ	0.53	0.03	0.1	80	Ψ	0.53	0.03	0.1	80
Ψ	0.48	0.02	0.2	80	Ψ	0.49	0.02	0.2	80	Ψ	0.49	0.02	0.2	80
Ψ	0.47	0.02	0.3	80	Ψ	0.48	0.02	0.3	80	Ψ	0.48	0.02	0.3	80
Ψ	0.48	0.02	0.4	80	Ψ	0.50	0.03	0.4	80	Ψ	0.50	0.03	0.4	80
Ψ	0.51	0.03	0.8	80	Ψ	0.52	0.03	0.8	80	Ψ	0.52	0.03	0.8	80

Table B.6. Probit, Ψ=1.0

Ψ=1.0	Model A					Model B					Model C			
	Estimate	SE	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj
B0	0.94	0.04	0.1	5	B0	0.95	0.04	0.1	5	B0	0.95	0.04	0.1	5
B0	0.94	0.05	0.2	5	B0	0.94	0.05	0.2	5	B0	0.92	0.05	0.2	5
B0	1.05	0.05	0.3	5	B0	1.05	0.05	0.3	5	B0	1.05	0.05	0.3	5
B0	0.98	0.05	0.4	5	B0	0.96	0.05	0.4	5	B0	0.94	0.05	0.4	5
B0	0.96	0.05	0.8	5	B0	0.96	0.05	0.8	5	B0	0.96	0.06	0.8	5
B0	0.99	0.04	0.1	10	B0	0.99	0.04	0.1	10	B0	0.99	0.04	0.1	10
B0	1.01	0.04	0.2	10	B0	1.01	0.04	0.2	10	B0	1.02	0.04	0.2	10
B0	1.00	0.04	0.3	10	B0	1.00	0.04	0.3	10	B0	0.99	0.04	0.3	10
B0	0.99	0.04	0.4	10	B0	1.00	0.04	0.4	10	B0	1.00	0.04	0.4	10
B0	1.06	0.04	0.8	10	B0	1.04	0.05	0.8	10	B0	1.04	0.05	0.8	10
B0	1.01	0.04	0.1	20	B0	1.01	0.04	0.1	20	B0	1.01	0.04	0.1	20
B0	1.00	0.04	0.2	20	B0	1.00	0.04	0.2	20	B0	1.00	0.04	0.2	20
B0	1.01	0.04	0.3	20	B0	1.00	0.04	0.3	20	B0	1.00	0.04	0.3	20
B0	1.03	0.04	0.4	20	B0	1.03	0.04	0.4	20	B0	1.04	0.04	0.4	20
B0	1.04	0.04	0.8	20	B0	1.04	0.04	0.8	20	B0	1.04	0.04	0.8	20
B0	1.02	0.03	0.1	40	B0	1.02	0.03	0.1	40	B0	1.02	0.03	0.1	40

B0	1.02	0.03	0.2	40	B0	1.02	0.03	0.2	40	B0	1.02	0.03	0.2	40
B0	1.03	0.04	0.3	40	B0	1.03	0.04	0.3	40	B0	1.03	0.04	0.3	40
B0	1.05	0.03	0.4	40	B0	1.04	0.03	0.4	40	B0	1.04	0.03	0.4	40
B0	1.04	0.04	0.8	40	B0	1.05	0.04	0.8	40	B0	1.05	0.04	0.8	40
B0	1.00	0.03	0.1	80	B0	1.00	0.03	0.1	80	B0	1.00	0.03	0.1	80
B0	1.02	0.03	0.2	80	B0	1.02	0.03	0.2	80	B0	1.02	0.03	0.2	80
B0	0.99	0.03	0.3	80	B0	0.99	0.03	0.3	80	B0	0.98	0.03	0.3	80
B0	0.99	0.03	0.4	80	B0	0.99	0.03	0.4	80	B0	0.99	0.03	0.4	80
B0	1.00	0.03	0.8	80	B0	1.00	0.03	0.8	80	B0	1.00	0.03	0.8	80
B1	0.99	0.04	0.1	5	B1	1.01	0.04	0.1	5	B1	1.01	0.04	0.1	5
B1	1.08	0.04	0.2	5	B1	1.08	0.04	0.2	5	B1	1.08	0.04	0.2	5
B1	0.99	0.04	0.3	5	B1	0.99	0.04	0.3	5	B1	0.99	0.04	0.3	5
B1	1.05	0.04	0.4	5	B1	1.04	0.04	0.4	5	B1	1.04	0.04	0.4	5
B1	0.99	0.04	0.8	5	B1	0.99	0.04	0.8	5	B1	0.99	0.04	0.8	5
B1	0.99	0.02	0.1	10	B1	0.99	0.02	0.1	10	B1	0.99	0.02	0.1	10
B1	1.03	0.03	0.2	10	B1	1.03	0.03	0.2	10	B1	1.03	0.03	0.2	10
B1	1.00	0.03	0.3	10	B1	1.00	0.03	0.3	10	B1	1.00	0.03	0.3	10
B1	1.03	0.03	0.4	10	B1	1.03	0.03	0.4	10	B1	1.03	0.03	0.4	10
B1	0.97	0.03	0.8	10	B1	0.97	0.03	0.8	10	B1	0.97	0.03	0.8	10
B1	0.98	0.02	0.1	20	B1	0.98	0.02	0.1	20	B1	0.98	0.02	0.1	20
B1	0.99	0.02	0.2	20	B1	0.99	0.02	0.2	20	B1	0.99	0.02	0.2	20
B1	0.96	0.02	0.3	20	B1	0.96	0.02	0.3	20	B1	0.96	0.02	0.3	20
B1	0.99	0.02	0.4	20	B1	0.99	0.02	0.4	20	B1	0.99	0.02	0.4	20
B1	1.02	0.02	0.8	20	B1	1.02	0.02	0.8	20	B1	1.02	0.02	0.8	20
B1	1.00	0.01	0.1	40	B1	1.00	0.01	0.1	40	B1	1.00	0.01	0.1	40
B1	1.00	0.01	0.2	40	B1	1.00	0.01	0.2	40	B1	1.00	0.01	0.2	40
B1	0.99	0.01	0.3	40	B1	0.99	0.01	0.3	40	B1	0.99	0.01	0.3	40
B1	1.00	0.01	0.4	40	B1	1.00	0.01	0.4	40	B1	1.00	0.01	0.4	40
B1	1.01	0.01	0.8	40	B1	1.01	0.01	0.8	40	B1	1.01	0.01	0.8	40
B1	0.99	0.01	0.1	80	B1	0.99	0.01	0.1	80	B1	0.99	0.01	0.1	80
B1	1.02	0.01	0.2	80	B1	1.02	0.01	0.2	80	B1	1.02	0.01	0.2	80
B1	1.01	0.01	0.3	80	B1	1.01	0.01	0.3	80	B1	1.01	0.01	0.3	80
B1	0.98	0.01	0.4	80	B1	0.98	0.01	0.4	80	B1	0.98	0.01	0.4	80

B1	1.01	0.01	0.8	80	B1	1.01	0.01	0.8	80	B1	1.01	0.01	0.8	80
B2	2.45	0.38	0.1	5	B2	0.96	0.09	0.1	5	B2	0.27	1.44	0.1	5
B2	2.06	0.22	0.2	5	B2	1.23	0.09	0.2	5	B2	1.90	0.50	0.2	5
B2	2.01	0.15	0.3	5	B2	1.29	0.08	0.3	5	B2	1.97	0.26	0.3	5
B2	1.92	0.12	0.4	5	B2	1.47	0.08	0.4	5	B2	2.04	0.18	0.4	5
B2	2.01	0.09	0.8	5	B2	1.77	0.08	0.8	5	B2	2.01	0.10	0.8	5
B2	1.68	0.39	0.1	10	B2	1.01	0.11	0.1	10	B2	1.10	0.79	0.1	10
B2	1.82	0.18	0.2	10	B2	1.36	0.10	0.2	10	B2	2.11	0.33	0.2	10
B2	2.19	0.13	0.3	10	B2	1.50	0.09	0.3	10	B2	2.10	0.21	0.3	10
B2	2.17	0.11	0.4	10	B2	1.77	0.08	0.4	10	B2	2.18	0.13	0.4	10
B2	2.03	0.07	0.8	10	B2	1.89	0.07	0.8	10	B2	2.04	0.07	0.8	10
B2	1.64	0.34	0.1	20	B2	1.03	0.14	0.1	20	B2	1.23	0.76	0.1	20
B2	1.77	0.19	0.2	20	B2	1.27	0.13	0.2	20	B2	1.68	0.31	0.2	20
B2	1.92	0.12	0.3	20	B2	1.54	0.09	0.3	20	B2	1.87	0.15	0.3	20
B2	2.09	0.09	0.4	20	B2	1.76	0.08	0.4	20	B2	2.01	0.11	0.4	20
B2	2.08	0.06	0.8	20	B2	2.04	0.06	0.8	20	B2	2.12	0.06	0.8	20
B2	1.96	0.33	0.1	40	B2	1.24	0.18	0.1	40	B2	1.89	0.66	0.1	40
B2	2.03	0.17	0.2	40	B2	1.65	0.13	0.2	40	B2	2.06	0.22	0.2	40
B2	2.01	0.12	0.3	40	B2	1.81	0.11	0.3	40	B2	2.05	0.14	0.3	40
B2	2.05	0.09	0.4	40	B2	1.97	0.08	0.4	40	B2	2.13	0.10	0.4	40
B2	2.05	0.05	0.8	40	B2	2.04	0.05	0.8	40	B2	2.08	0.05	0.8	40
B2	2.10	0.33	0.1	80	B2	1.52	0.22	0.1	80	B2	2.22	0.51	0.1	80
B2	2.38	0.18	0.2	80	B2	2.02	0.15	0.2	80	B2	2.36	0.20	0.2	80
B2	2.22	0.11	0.3	80	B2	2.07	0.11	0.3	80	B2	2.22	0.12	0.3	80
B2	2.05	0.09	0.4	80	B2	1.99	0.08	0.4	80	B2	2.06	0.09	0.4	80
B2	2.03	0.05	0.8	80	B2	2.00	0.05	0.8	80	B2	2.02	0.05	0.8	80
Ψ	0.83	0.09	0.1	5	Ψ	0.85	0.09	0.1	5	Ψ	0.85	0.09	0.1	5
Ψ	1.09	0.11	0.2	5	Ψ	1.11	0.11	0.2	5	Ψ	1.11	0.11	0.2	5
Ψ	0.90	0.10	0.3	5	Ψ	0.96	0.10	0.3	5	Ψ	0.96	0.10	0.3	5
Ψ	0.96	0.10	0.4	5	Ψ	1.02	0.11	0.4	5	Ψ	1.02	0.11	0.4	5
Ψ	1.05	0.12	0.8	5	Ψ	1.21	0.13	0.8	5	Ψ	1.21	0.13	0.8	5
Ψ	1.00	0.07	0.1	10	Ψ	1.01	0.07	0.1	10	Ψ	1.01	0.07	0.1	10
Ψ	1.00	0.08	0.2	10	Ψ	1.01	0.08	0.2	10	Ψ	1.01	0.08	0.2	10

Ψ	1.02	0.08	0.3	10	Ψ	1.10	0.08	0.3	10	Ψ	1.10	0.08	0.3	10
Ψ	1.11	0.08	0.4	10	Ψ	1.18	0.09	0.4	10	Ψ	1.18	0.09	0.4	10
Ψ	0.97	0.08	0.8	10	Ψ	1.06	0.09	0.8	10	Ψ	1.06	0.09	0.8	10
Ψ	1.05	0.06	0.1	20	Ψ	1.06	0.06	0.1	20	Ψ	1.06	0.06	0.1	20
Ψ	1.12	0.07	0.2	20	Ψ	1.14	0.07	0.2	20	Ψ	1.14	0.07	0.2	20
Ψ	0.95	0.06	0.3	20	Ψ	0.98	0.06	0.3	20	Ψ	0.98	0.06	0.3	20
Ψ	1.02	0.06	0.4	20	Ψ	1.09	0.07	0.4	20	Ψ	1.09	0.07	0.4	20
Ψ	1.09	0.07	0.8	20	Ψ	1.10	0.07	0.8	20	Ψ	1.10	0.07	0.8	20
Ψ	0.95	0.05	0.1	40	Ψ	0.96	0.05	0.1	40	Ψ	0.96	0.05	0.1	40
Ψ	1.00	0.05	0.2	40	Ψ	1.01	0.05	0.2	40	Ψ	1.01	0.05	0.2	40
Ψ	1.09	0.06	0.3	40	Ψ	1.10	0.06	0.3	40	Ψ	1.10	0.06	0.3	40
Ψ	0.99	0.05	0.4	40	Ψ	1.00	0.05	0.4	40	Ψ	1.00	0.05	0.4	40
Ψ	0.99	0.06	0.8	40	Ψ	1.01	0.06	0.8	40	Ψ	1.01	0.06	0.8	40
Ψ	1.03	0.05	0.1	80	Ψ	1.04	0.05	0.1	80	Ψ	1.04	0.05	0.1	80
Ψ	1.08	0.05	0.2	80	Ψ	1.09	0.05	0.2	80	Ψ	1.09	0.05	0.2	80
Ψ	1.02	0.05	0.3	80	Ψ	1.04	0.05	0.3	80	Ψ	1.04	0.05	0.3	80
Ψ	1.09	0.06	0.4	80	Ψ	1.10	0.06	0.4	80	Ψ	1.10	0.06	0.4	80
Ψ	1.01	0.05	0.8	80	Ψ	1.03	0.06	0.8	80	Ψ	1.03	0.06	0.8	80

3. Poisson Simulations

Table B.7. Poisson, Ψ=0.2

Ψ=0.2	Model A					Model B					Model C			
	Estimate	SE	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj
B0	1.00	0.02	0.1	5	B0	1.00	0.02	0.1	5	B0	1.00	0.02	0.1	5
B0	0.98	0.02	0.2	5	B0	0.98	0.02	0.2	5	B0	0.98	0.02	0.2	5
B0	0.98	0.02	0.3	5	B0	0.99	0.02	0.3	5	B0	1.02	0.02	0.3	5
B0	1.01	0.02	0.4	5	B0	1.02	0.02	0.4	5	B0	1.01	0.02	0.4	5
B0	1.01	0.02	0.8	5	B0	1.03	0.02	0.8	5	B0	1.04	0.02	0.8	5
B0	1.00	0.02	0.1	10	B0	1.00	0.02	0.1	10	B0	0.98	0.02	0.1	10
B0	1.01	0.02	0.2	10	B0	1.01	0.02	0.2	10	B0	1.01	0.02	0.2	10
B0	1.00	0.02	0.3	10	B0	1.02	0.02	0.3	10	B0	1.02	0.02	0.3	10
B0	0.97	0.02	0.4	10	B0	0.98	0.02	0.4	10	B0	0.97	0.02	0.4	10
B0	1.01	0.02	0.8	10	B0	1.00	0.02	0.8	10	B0	1.00	0.02	0.8	10

B0	1.00	0.01	0.1	20	B0	1.00	0.02	0.1	20	B0	1.01	0.02	0.1	20
B0	0.98	0.02	0.2	20	B0	0.98	0.02	0.2	20	B0	0.99	0.02	0.2	20
B0	0.99	0.01	0.3	20	B0	0.99	0.02	0.3	20	B0	0.98	0.02	0.3	20
B0	0.99	0.01	0.4	20	B0	1.00	0.02	0.4	20	B0	0.99	0.02	0.4	20
B0	1.02	0.02	0.8	20	B0	1.01	0.02	0.8	20	B0	1.01	0.02	0.8	20
B0	1.03	0.01	0.1	40	B0	1.03	0.02	0.1	40	B0	1.02	0.02	0.1	40
B0	1.00	0.01	0.2	40	B0	1.00	0.01	0.2	40	B0	1.00	0.01	0.2	40
B0	0.99	0.01	0.3	40	B0	0.99	0.02	0.3	40	B0	1.00	0.02	0.3	40
B0	1.03	0.01	0.4	40	B0	1.02	0.02	0.4	40	B0	1.03	0.02	0.4	40
B0	1.01	0.01	0.8	40	B0	1.01	0.02	0.8	40	B0	1.01	0.02	0.8	40
B0	0.99	0.01	0.1	80	B0	0.98	0.01	0.1	80	B0	0.99	0.01	0.1	80
B0	1.03	0.01	0.2	80	B0	1.03	0.01	0.2	80	B0	1.03	0.01	0.2	80
B0	0.99	0.01	0.3	80	B0	0.98	0.01	0.3	80	B0	0.98	0.02	0.3	80
B0	1.00	0.01	0.4	80	B0	1.00	0.01	0.4	80	B0	1.00	0.01	0.4	80
B0	0.99	0.01	0.8	80	B0	0.98	0.02	0.8	80	B0	0.98	0.02	0.8	80
B1	1.01	0.01	0.1	5	B1	1.01	0.01	0.1	5	B1	1.01	0.01	0.1	5
B1	0.99	0.01	0.2	5	B1	0.99	0.01	0.2	5	B1	0.99	0.01	0.2	5
B1	1.00	0.01	0.3	5	B1	0.99	0.01	0.3	5	B1	0.99	0.01	0.3	5
B1	1.01	0.01	0.4	5	B1	1.01	0.01	0.4	5	B1	1.01	0.01	0.4	5
B1	1.00	0.01	0.8	5	B1	1.00	0.01	0.8	5	B1	1.00	0.01	0.8	5
B1	1.00	0.01	0.1	10	B1	1.00	0.01	0.1	10	B1	1.00	0.01	0.1	10
B1	1.00	0.01	0.2	10	B1	1.00	0.01	0.2	10	B1	1.00	0.01	0.2	10
B1	1.00	0.00	0.3	10	B1	1.00	0.00	0.3	10	B1	1.00	0.00	0.3	10
B1	1.00	0.00	0.4	10	B1	1.00	0.00	0.4	10	B1	1.00	0.00	0.4	10
B1	1.00	0.00	0.8	10	B1	1.00	0.00	0.8	10	B1	1.00	0.00	0.8	10
B1	1.00	0.00	0.1	20	B1	1.00	0.00	0.1	20	B1	1.00	0.00	0.1	20
B1	1.00	0.00	0.2	20	B1	1.00	0.00	0.2	20	B1	1.00	0.00	0.2	20
B1	1.00	0.00	0.3	20	B1	1.00	0.00	0.3	20	B1	1.00	0.00	0.3	20
B1	1.00	0.00	0.4	20	B1	1.00	0.00	0.4	20	B1	1.00	0.00	0.4	20
B1	1.00	0.00	0.8	20	B1	1.00	0.00	0.8	20	B1	1.00	0.00	0.8	20
B1	1.00	0.00	0.1	40	B1	1.00	0.00	0.1	40	B1	1.00	0.00	0.1	40
B1	1.00	0.00	0.2	40	B1	1.00	0.00	0.2	40	B1	1.00	0.00	0.2	40
B1	1.00	0.00	0.3	40	B1	1.00	0.00	0.3	40	B1	1.00	0.00	0.3	40

B1	1.00	0.00	0.4	40	B1	1.00	0.00	0.4	40	B1	1.00	0.00	0.4	40
B1	1.00	0.00	0.8	40	B1	1.00	0.00	0.8	40	B1	1.00	0.00	0.8	40
B1	1.00	0.00	0.1	80	B1	1.00	0.00	0.1	80	B1	1.00	0.00	0.1	80
B1	1.00	0.00	0.2	80	B1	1.00	0.00	0.2	80	B1	1.00	0.00	0.2	80
B1	1.00	0.00	0.3	80	B1	1.00	0.00	0.3	80	B1	1.00	0.00	0.3	80
B1	1.00	0.00	0.4	80	B1	1.00	0.00	0.4	80	B1	1.00	0.00	0.4	80
B1	1.00	0.00	0.8	80	B1	1.00	0.00	0.8	80	B1	1.00	0.00	0.8	80
B2	2.11	0.17	0.1	5	B2	1.02	0.04	0.1	5	B2	1.20	0.50	0.1	5
B2	1.85	0.08	0.2	5	B2	1.19	0.03	0.2	5	B2	1.89	0.15	0.2	5
B2	2.02	0.06	0.3	5	B2	1.33	0.04	0.3	5	B2	2.18	0.12	0.3	5
B2	2.03	0.04	0.4	5	B2	1.44	0.03	0.4	5	B2	2.00	0.08	0.4	5
B2	2.01	0.03	0.8	5	B2	1.75	0.03	0.8	5	B2	2.01	0.04	0.8	5
B2	2.05	0.15	0.1	10	B2	1.12	0.05	0.1	10	B2	2.63	0.66	0.1	10
B2	1.95	0.08	0.2	10	B2	1.31	0.04	0.2	10	B2	1.99	0.14	0.2	10
B2	1.94	0.05	0.3	10	B2	1.41	0.04	0.3	10	B2	1.83	0.07	0.3	10
B2	1.99	0.04	0.4	10	B2	1.63	0.03	0.4	10	B2	2.03	0.05	0.4	10
B2	1.99	0.02	0.8	10	B2	1.85	0.02	0.8	10	B2	1.99	0.03	0.8	10
B2	1.98	0.14	0.1	20	B2	1.12	0.06	0.1	20	B2	2.34	0.69	0.1	20
B2	1.95	0.08	0.2	20	B2	1.38	0.05	0.2	20	B2	1.87	0.12	0.2	20
B2	2.04	0.05	0.3	20	B2	1.68	0.04	0.3	20	B2	2.04	0.06	0.3	20
B2	1.95	0.04	0.4	20	B2	1.71	0.04	0.4	20	B2	1.93	0.05	0.4	20
B2	1.98	0.02	0.8	20	B2	1.90	0.02	0.8	20	B2	1.97	0.02	0.8	20
B2	2.19	0.14	0.1	40	B2	1.31	0.08	0.1	40	B2	2.04	0.27	0.1	40
B2	2.10	0.07	0.2	40	B2	1.67	0.06	0.2	40	B2	2.07	0.09	0.2	40
B2	2.04	0.05	0.3	40	B2	1.83	0.04	0.3	40	B2	2.05	0.06	0.3	40
B2	2.03	0.04	0.4	40	B2	1.90	0.04	0.4	40	B2	2.03	0.04	0.4	40
B2	2.01	0.02	0.8	40	B2	1.98	0.02	0.8	40	B2	2.02	0.02	0.8	40
B2	1.96	0.14	0.1	80	B2	1.38	0.10	0.1	80	B2	1.86	0.22	0.1	80
B2	1.98	0.07	0.2	80	B2	1.75	0.06	0.2	80	B2	1.99	0.08	0.2	80
B2	2.08	0.05	0.3	80	B2	1.95	0.05	0.3	80	B2	2.09	0.05	0.3	80
B2	1.96	0.04	0.4	80	B2	1.88	0.04	0.4	80	B2	1.95	0.04	0.4	80
B2	2.00	0.02	0.8	80	B2	1.98	0.02	0.8	80	B2	2.00	0.02	0.8	80
Ψ	0.23	0.01	0.1	5	Ψ	0.24	0.01	0.1	5	Ψ	0.24	0.01	0.1	5

Ψ	0.17	0.01	0.2	5	Ψ	0.19	0.01	0.2	5	Ψ	0.19	0.01	0.2	5
Ψ	0.20	0.01	0.3	5	Ψ	0.26	0.01	0.3	5	Ψ	0.26	0.01	0.3	5
Ψ	0.20	0.01	0.4	5	Ψ	0.30	0.02	0.4	5	Ψ	0.30	0.02	0.4	5
Ψ	0.21	0.01	0.8	5	Ψ	0.35	0.02	0.8	5	Ψ	0.35	0.02	0.8	5
Ψ	0.21	0.01	0.1	10	Ψ	0.22	0.01	0.1	10	Ψ	0.22	0.01	0.1	10
Ψ	0.20	0.01	0.2	10	Ψ	0.23	0.01	0.2	10	Ψ	0.23	0.01	0.2	10
Ψ	0.18	0.01	0.3	10	Ψ	0.23	0.01	0.3	10	Ψ	0.23	0.01	0.3	10
Ψ	0.18	0.01	0.4	10	Ψ	0.24	0.01	0.4	10	Ψ	0.24	0.01	0.4	10
Ψ	0.19	0.01	0.8	10	Ψ	0.29	0.02	0.8	10	Ψ	0.29	0.02	0.8	10
Ψ	0.20	0.01	0.1	20	Ψ	0.21	0.01	0.1	20	Ψ	0.21	0.01	0.1	20
Ψ	0.21	0.01	0.2	20	Ψ	0.23	0.01	0.2	20	Ψ	0.23	0.01	0.2	20
Ψ	0.19	0.01	0.3	20	Ψ	0.22	0.01	0.3	20	Ψ	0.22	0.01	0.3	20
Ψ	0.20	0.01	0.4	20	Ψ	0.23	0.01	0.4	20	Ψ	0.23	0.01	0.4	20
Ψ	0.20	0.01	0.8	20	Ψ	0.25	0.01	0.8	20	Ψ	0.25	0.01	0.8	20
Ψ	0.21	0.01	0.1	40	Ψ	0.22	0.01	0.1	40	Ψ	0.22	0.01	0.1	40
Ψ	0.19	0.01	0.2	40	Ψ	0.21	0.01	0.2	40	Ψ	0.21	0.01	0.2	40
Ψ	0.21	0.01	0.3	40	Ψ	0.23	0.01	0.3	40	Ψ	0.23	0.01	0.3	40
Ψ	0.20	0.01	0.4	40	Ψ	0.22	0.01	0.4	40	Ψ	0.22	0.01	0.4	40
Ψ	0.19	0.01	0.8	40	Ψ	0.21	0.01	0.8	40	Ψ	0.21	0.01	0.8	40
Ψ	0.20	0.01	0.1	80	Ψ	0.20	0.01	0.1	80	Ψ	0.20	0.01	0.1	80
Ψ	0.20	0.01	0.2	80	Ψ	0.21	0.01	0.2	80	Ψ	0.21	0.01	0.2	80
Ψ	0.21	0.01	0.3	80	Ψ	0.22	0.01	0.3	80	Ψ	0.22	0.01	0.3	80
Ψ	0.20	0.01	0.4	80	Ψ	0.21	0.01	0.4	80	Ψ	0.21	0.01	0.4	80
Ψ	0.20	0.01	0.8	80	Ψ	0.21	0.01	0.8	80	Ψ	0.21	0.01	0.8	80

Table B.8. Poisson, Ψ=0.5

Ψ=0.5	Model A					Model B					Model C		
	Estimate	SE	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj
B0	0.99	0.02	0.1	5	B0	0.99	0.03	0.1	5	B0	0.98	0.03	0.1	5
B0	0.99	0.02	0.2	5	B0	1.00	0.03	0.2	5	B0	0.99	0.03	0.2	5
B0	0.98	0.03	0.3	5	B0	0.97	0.03	0.3	5	B0	0.97	0.03	0.3	5
B0	0.99	0.02	0.4	5	B0	1.00	0.03	0.4	5	B0	0.99	0.03	0.4	5
B0	1.04	0.03	0.8	5	B0	1.00	0.03	0.8	5	B0	1.00	0.03	0.8	5
B0	1.04	0.02	0.1	10	B0	1.04	0.02	0.1	10	B0	1.04	0.02	0.1	10
B0	0.98	0.02	0.2	10	B0	0.98	0.02	0.2	10	B0	0.98	0.02	0.2	10
B0	1.02	0.02	0.3	10	B0	1.03	0.02	0.3	10	B0	1.02	0.02	0.3	10
B0	1.01	0.02	0.4	10	B0	1.02	0.02	0.4	10	B0	1.03	0.02	0.4	10
B0	1.00	0.02	0.8	10	B0	1.02	0.03	0.8	10	B0	1.02	0.03	0.8	10
B0	1.00	0.02	0.1	20	B0	1.00	0.02	0.1	20	B0	1.01	0.02	0.1	20
B0	0.99	0.02	0.2	20	B0	0.98	0.02	0.2	20	B0	0.98	0.02	0.2	20
B0	1.00	0.02	0.3	20	B0	0.99	0.02	0.3	20	B0	0.99	0.02	0.3	20
B0	0.96	0.02	0.4	20	B0	0.97	0.02	0.4	20	B0	0.97	0.02	0.4	20
B0	1.00	0.02	0.8	20	B0	1.02	0.02	0.8	20	B0	1.02	0.02	0.8	20
B0	1.02	0.02	0.1	40	B0	1.03	0.02	0.1	40	B0	1.02	0.02	0.1	40
B0	1.01	0.02	0.2	40	B0	1.02	0.02	0.2	40	B0	1.02	0.02	0.2	40
B0	1.02	0.02	0.3	40	B0	1.02	0.02	0.3	40	B0	1.01	0.02	0.3	40
B0	0.99	0.02	0.4	40	B0	0.99	0.02	0.4	40	B0	0.99	0.02	0.4	40
B0	1.02	0.02	0.8	40	B0	1.02	0.02	0.8	40	B0	1.02	0.02	0.8	40
B0	0.98	0.02	0.1	80	B0	0.98	0.02	0.1	80	B0	0.99	0.02	0.1	80
B0	0.98	0.02	0.2	80	B0	0.98	0.02	0.2	80	B0	0.97	0.02	0.2	80
B0	0.98	0.02	0.3	80	B0	0.98	0.02	0.3	80	B0	0.98	0.02	0.3	80
B0	0.97	0.02	0.4	80	B0	0.97	0.02	0.4	80	B0	0.97	0.02	0.4	80
B0	0.98	0.02	0.8	80	B0	0.98	0.02	0.8	80	B0	0.98	0.02	0.8	80
B1	1.00	0.01	0.1	5	B1	1.00	0.01	0.1	5	B1	1.00	0.01	0.1	5
B1	0.99	0.01	0.2	5	B1	0.99	0.01	0.2	5	B1	0.99	0.01	0.2	5
B1	0.99	0.01	0.3	5	B1	0.99	0.01	0.3	5	B1	0.99	0.01	0.3	5
B1	1.00	0.01	0.4	5	B1	1.00	0.01	0.4	5	B1	1.00	0.01	0.4	5

B1	1.00	0.00	0.8	5	B1	1.00	0.00	0.8	5	B1	1.00	0.00	0.8	5
B1	1.00	0.00	0.1	10	B1	1.00	0.00	0.1	10	B1	1.00	0.00	0.1	10
B1	1.00	0.00	0.2	10	B1	1.00	0.00	0.2	10	B1	1.00	0.00	0.2	10
B1	1.00	0.00	0.3	10	B1	1.00	0.00	0.3	10	B1	1.00	0.00	0.3	10
B1	1.00	0.00	0.4	10	B1	1.00	0.00	0.4	10	B1	1.00	0.00	0.4	10
B1	1.00	0.00	0.8	10	B1	1.00	0.00	0.8	10	B1	1.00	0.00	0.8	10
B1	1.00	0.00	0.1	20	B1	1.00	0.00	0.1	20	B1	1.00	0.00	0.1	20
B1	1.00	0.00	0.2	20	B1	1.00	0.00	0.2	20	B1	1.00	0.00	0.2	20
B1	1.00	0.00	0.3	20	B1	1.00	0.00	0.3	20	B1	1.00	0.00	0.3	20
B1	1.00	0.00	0.4	20	B1	1.00	0.00	0.4	20	B1	1.00	0.00	0.4	20
B1	1.00	0.00	0.8	20	B1	1.00	0.00	0.8	20	B1	1.00	0.00	0.8	20
B1	1.00	0.00	0.1	40	B1	1.00	0.00	0.1	40	B1	1.00	0.00	0.1	40
B1	1.00	0.00	0.2	40	B1	1.00	0.00	0.2	40	B1	1.00	0.00	0.2	40
B1	1.00	0.00	0.3	40	B1	1.00	0.00	0.3	40	B1	1.00	0.00	0.3	40
B1	1.00	0.00	0.4	40	B1	1.00	0.00	0.4	40	B1	1.00	0.00	0.4	40
B1	1.00	0.00	0.8	40	B1	1.00	0.00	0.8	40	B1	1.00	0.00	0.8	40
B1	1.00	0.00	0.1	80	B1	1.00	0.00	0.1	80	B1	1.00	0.00	0.1	80
B1	1.00	0.00	0.2	80	B1	1.00	0.00	0.2	80	B1	1.00	0.00	0.2	80
B1	1.00	0.00	0.3	80	B1	1.00	0.00	0.3	80	B1	1.00	0.00	0.3	80
B1	1.00	0.00	0.4	80	B1	1.00	0.00	0.4	80	B1	1.00	0.00	0.4	80
B1	1.00	0.00	0.8	80	B1	1.00	0.00	0.8	80	B1	1.00	0.00	0.8	80
B2	2.23	0.24	0.1	5	B2	1.04	0.05	0.1	5	B2	3.18	2.89	0.1	5
B2	2.18	0.12	0.2	5	B2	1.17	0.05	0.2	5	B2	2.01	0.28	0.2	5
B2	1.95	0.08	0.3	5	B2	1.27	0.05	0.3	5	B2	1.96	0.17	0.3	5
B2	1.88	0.06	0.4	5	B2	1.38	0.04	0.4	5	B2	1.86	0.09	0.4	5
B2	1.92	0.03	0.8	5	B2	1.72	0.03	0.8	5	B2	1.96	0.05	0.8	5
B2	1.93	0.23	0.1	10	B2	1.04	0.07	0.1	10	B2	1.34	0.57	0.1	10
B2	1.83	0.11	0.2	10	B2	1.22	0.06	0.2	10	B2	1.83	0.23	0.2	10
B2	1.89	0.08	0.3	10	B2	1.44	0.06	0.3	10	B2	2.04	0.14	0.3	10
B2	1.97	0.06	0.4	10	B2	1.54	0.05	0.4	10	B2	1.87	0.08	0.4	10
B2	2.03	0.03	0.8	10	B2	1.88	0.03	0.8	10	B2	2.01	0.04	0.8	10
B2	1.71	0.23	0.1	20	B2	1.21	0.09	0.1	20	B2	2.02	0.43	0.1	20
B2	2.06	0.12	0.2	20	B2	1.42	0.08	0.2	20	B2	1.89	0.17	0.2	20

B2	2.06	0.08	0.3	20	B2	1.63	0.06	0.3	20	B2	2.00	0.10	0.3	20
B2	2.04	0.06	0.4	20	B2	1.82	0.05	0.4	20	B2	2.07	0.06	0.4	20
B2	1.98	0.03	0.8	20	B2	1.87	0.03	0.8	20	B2	1.94	0.03	0.8	20
B2	1.99	0.23	0.1	40	B2	1.41	0.12	0.1	40	B2	2.35	0.41	0.1	40
B2	2.23	0.11	0.2	40	B2	1.83	0.08	0.2	40	B2	2.29	0.13	0.2	40
B2	2.15	0.08	0.3	40	B2	1.91	0.07	0.3	40	B2	2.15	0.09	0.3	40
B2	1.95	0.06	0.4	40	B2	1.83	0.06	0.4	40	B2	1.97	0.07	0.4	40
B2	2.03	0.03	0.8	40	B2	2.00	0.03	0.8	40	B2	2.04	0.03	0.8	40
B2	2.19	0.23	0.1	80	B2	1.51	0.15	0.1	80	B2	2.15	0.35	0.1	80
B2	2.25	0.11	0.2	80	B2	2.01	0.10	0.2	80	B2	2.36	0.14	0.2	80
B2	1.96	0.07	0.3	80	B2	1.81	0.07	0.3	80	B2	1.92	0.08	0.3	80
B2	1.96	0.05	0.4	80	B2	1.87	0.05	0.4	80	B2	1.93	0.06	0.4	80
B2	1.99	0.03	0.8	80	B2	1.97	0.03	0.8	80	B2	1.99	0.03	0.8	80
Ψ	0.52	0.03	0.1	5	Ψ	0.53	0.03	0.1	5	Ψ	0.53	0.03	0.1	5
Ψ	0.51	0.03	0.2	5	Ψ	0.56	0.03	0.2	5	Ψ	0.56	0.03	0.2	5
Ψ	0.55	0.03	0.3	5	Ψ	0.61	0.03	0.3	5	Ψ	0.61	0.03	0.3	5
Ψ	0.47	0.02	0.4	5	Ψ	0.54	0.03	0.4	5	Ψ	0.54	0.03	0.4	5
Ψ	0.50	0.03	0.8	5	Ψ	0.66	0.04	0.8	5	Ψ	0.66	0.04	0.8	5
Ψ	0.46	0.02	0.1	10	Ψ	0.47	0.02	0.1	10	Ψ	0.47	0.02	0.1	10
Ψ	0.46	0.02	0.2	10	Ψ	0.49	0.02	0.2	10	Ψ	0.49	0.02	0.2	10
Ψ	0.51	0.02	0.3	10	Ψ	0.55	0.03	0.3	10	Ψ	0.55	0.03	0.3	10
Ψ	0.48	0.02	0.4	10	Ψ	0.53	0.03	0.4	10	Ψ	0.53	0.03	0.4	10
Ψ	0.49	0.02	0.8	10	Ψ	0.57	0.03	0.8	10	Ψ	0.57	0.03	0.8	10
Ψ	0.51	0.02	0.1	20	Ψ	0.51	0.02	0.1	20	Ψ	0.51	0.02	0.1	20
Ψ	0.54	0.03	0.2	20	Ψ	0.57	0.03	0.2	20	Ψ	0.57	0.03	0.2	20
Ψ	0.49	0.02	0.3	20	Ψ	0.54	0.03	0.3	20	Ψ	0.54	0.03	0.3	20
Ψ	0.47	0.02	0.4	20	Ψ	0.50	0.02	0.4	20	Ψ	0.50	0.02	0.4	20
Ψ	0.46	0.02	0.8	20	Ψ	0.51	0.02	0.8	20	Ψ	0.51	0.02	0.8	20
Ψ	0.51	0.02	0.1	40	Ψ	0.52	0.02	0.1	40	Ψ	0.52	0.02	0.1	40
Ψ	0.47	0.02	0.2	40	Ψ	0.48	0.02	0.2	40	Ψ	0.48	0.02	0.2	40
Ψ	0.53	0.02	0.3	40	Ψ	0.56	0.03	0.3	40	Ψ	0.56	0.03	0.3	40
Ψ	0.51	0.02	0.4	40	Ψ	0.53	0.02	0.4	40	Ψ	0.53	0.02	0.4	40
Ψ	0.47	0.02	0.8	40	Ψ	0.50	0.02	0.8	40	Ψ	0.50	0.02	0.8	40

Ψ	0.52	0.02	0.1	80	Ψ	0.53	0.02	0.1	80	Ψ	0.53	0.02	0.1	80
Ψ	0.48	0.02	0.2	80	Ψ	0.50	0.02	0.2	80	Ψ	0.50	0.02	0.2	80
Ψ	0.46	0.02	0.3	80	Ψ	0.48	0.02	0.3	80	Ψ	0.48	0.02	0.3	80
Ψ	0.49	0.02	0.4	80	Ψ	0.50	0.02	0.4	80	Ψ	0.50	0.02	0.4	80
Ψ	0.50	0.02	0.8	80	Ψ	0.51	0.02	0.8	80	Ψ	0.51	0.02	0.8	80

Table B.9. Poisson, Ψ=1.0

Ψ=1.0	Model A					Model B					Model C			
	Estimate	SE	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj
B0	0.98	0.03	0.1	5	B0	0.98	0.03	0.1	5	B0	0.98	0.03	0.1	5
B0	1.00	0.03	0.2	5	B0	1.00	0.03	0.2	5	B0	0.97	0.04	0.2	5
B0	1.09	0.03	0.3	5	B0	1.05	0.03	0.3	5	B0	1.05	0.03	0.3	5
B0	0.98	0.03	0.4	5	B0	0.96	0.03	0.4	5	B0	0.95	0.03	0.4	5
B0	1.01	0.03	0.8	5	B0	1.01	0.04	0.8	5	B0	1.01	0.04	0.8	5
B0	1.03	0.03	0.1	10	B0	1.01	0.03	0.1	10	B0	1.01	0.03	0.1	10
B0	1.00	0.03	0.2	10	B0	1.00	0.03	0.2	10	B0	1.01	0.03	0.2	10
B0	1.01	0.03	0.3	10	B0	1.00	0.03	0.3	10	B0	1.00	0.03	0.3	10
B0	0.97	0.03	0.4	10	B0	0.98	0.03	0.4	10	B0	0.98	0.03	0.4	10
B0	1.08	0.03	0.8	10	B0	1.04	0.04	0.8	10	B0	1.04	0.04	0.8	10
B0	1.02	0.03	0.1	20	B0	1.01	0.03	0.1	20	B0	1.01	0.03	0.1	20
B0	0.99	0.03	0.2	20	B0	1.01	0.03	0.2	20	B0	1.01	0.03	0.2	20
B0	1.02	0.03	0.3	20	B0	1.01	0.03	0.3	20	B0	1.01	0.03	0.3	20
B0	1.04	0.03	0.4	20	B0	1.05	0.03	0.4	20	B0	1.05	0.03	0.4	20
B0	1.01	0.03	0.8	20	B0	1.00	0.03	0.8	20	B0	1.00	0.03	0.8	20
B0	1.04	0.03	0.1	40	B0	1.04	0.03	0.1	40	B0	1.04	0.03	0.1	40
B0	1.02	0.03	0.2	40	B0	1.02	0.03	0.2	40	B0	1.02	0.03	0.2	40
B0	1.03	0.03	0.3	40	B0	1.02	0.03	0.3	40	B0	1.02	0.03	0.3	40
B0	1.04	0.03	0.4	40	B0	1.04	0.03	0.4	40	B0	1.04	0.03	0.4	40
B0	1.05	0.03	0.8	40	B0	1.06	0.03	0.8	40	B0	1.06	0.03	0.8	40
B0	1.00	0.03	0.1	80	B0	0.99	0.03	0.1	80	B0	1.00	0.03	0.1	80
B0	1.01	0.03	0.2	80	B0	1.01	0.03	0.2	80	B0	1.01	0.03	0.2	80
B0	0.98	0.03	0.3	80	B0	0.98	0.03	0.3	80	B0	0.97	0.03	0.3	80

B0	0.98	0.03	0.4	80	B0	0.98	0.03	0.4	80	B0	0.98	0.03	0.4	80
B0	0.96	0.03	0.8	80	B0	0.96	0.03	0.8	80	B0	0.96	0.03	0.8	80
B1	1.00	0.01	0.1	5	B1	1.01	0.01	0.1	5	B1	1.01	0.01	0.1	5
B1	1.00	0.01	0.2	5	B1	1.00	0.01	0.2	5	B1	1.00	0.01	0.2	5
B1	1.00	0.01	0.3	5	B1	1.00	0.01	0.3	5	B1	1.00	0.01	0.3	5
B1	1.00	0.01	0.4	5	B1	1.00	0.01	0.4	5	B1	1.00	0.01	0.4	5
B1	1.01	0.00	0.8	5	B1	1.01	0.00	0.8	5	B1	1.01	0.00	0.8	5
B1	1.00	0.00	0.1	10	B1	1.00	0.00	0.1	10	B1	1.00	0.00	0.1	10
B1	1.00	0.00	0.2	10	B1	1.00	0.00	0.2	10	B1	1.00	0.00	0.2	10
B1	1.00	0.00	0.3	10	B1	1.00	0.00	0.3	10	B1	1.00	0.00	0.3	10
B1	1.00	0.00	0.4	10	B1	1.00	0.00	0.4	10	B1	1.00	0.00	0.4	10
B1	1.00	0.00	0.8	10	B1	1.00	0.00	0.8	10	B1	1.00	0.00	0.8	10
B1	1.00	0.00	0.1	20	B1	1.00	0.00	0.1	20	B1	1.00	0.00	0.1	20
B1	1.00	0.00	0.2	20	B1	1.00	0.00	0.2	20	B1	1.00	0.00	0.2	20
B1	1.00	0.00	0.3	20	B1	1.00	0.00	0.3	20	B1	1.00	0.00	0.3	20
B1	1.00	0.00	0.4	20	B1	1.00	0.00	0.4	20	B1	1.00	0.00	0.4	20
B1	1.00	0.00	0.8	20	B1	1.00	0.00	0.8	20	B1	1.00	0.00	0.8	20
B1	1.00	0.00	0.1	40	B1	1.00	0.00	0.1	40	B1	1.00	0.00	0.1	40
B1	1.00	0.00	0.2	40	B1	1.00	0.00	0.2	40	B1	1.00	0.00	0.2	40
B1	1.00	0.00	0.3	40	B1	1.00	0.00	0.3	40	B1	1.00	0.00	0.3	40
B1	1.00	0.00	0.4	40	B1	1.00	0.00	0.4	40	B1	1.00	0.00	0.4	40
B1	1.00	0.00	0.8	40	B1	1.00	0.00	0.8	40	B1	1.00	0.00	0.8	40
B1	1.00	0.00	0.1	80	B1	1.00	0.00	0.1	80	B1	1.00	0.00	0.1	80
B1	1.00	0.00	0.2	80	B1	1.00	0.00	0.2	80	B1	1.00	0.00	0.2	80
B1	1.00	0.00	0.3	80	B1	1.00	0.00	0.3	80	B1	1.00	0.00	0.3	80
B1	1.00	0.00	0.4	80	B1	1.00	0.00	0.4	80	B1	1.00	0.00	0.4	80
B1	1.00	0.00	0.8	80	B1	1.00	0.00	0.8	80	B1	1.00	0.00	0.8	80
B2	2.48	0.32	0.1	5	B2	1.02	0.07	0.1	5	B2	1.21	1.19	0.1	5
B2	2.02	0.17	0.2	5	B2	1.20	0.07	0.2	5	B2	2.08	0.38	0.2	5
B2	1.88	0.11	0.3	5	B2	1.21	0.06	0.3	5	B2	1.70	0.20	0.3	5
B2	1.82	0.08	0.4	5	B2	1.39	0.06	0.4	5	B2	1.91	0.14	0.4	5
B2	2.01	0.05	0.8	5	B2	1.80	0.04	0.8	5	B2	2.05	0.06	0.8	5
B2	1.76	0.33	0.1	10	B2	1.03	0.10	0.1	10	B2	1.22	0.69	0.1	10

B2	1.98	0.16	0.2	10	B2	1.39	0.09	0.2	10	B2	2.27	0.28	0.2	10
B2	2.06	0.11	0.3	10	B2	1.40	0.08	0.3	10	B2	1.89	0.17	0.3	10
B2	2.11	0.08	0.4	10	B2	1.73	0.06	0.4	10	B2	2.13	0.10	0.4	10
B2	1.99	0.04	0.8	10	B2	1.91	0.04	0.8	10	B2	2.05	0.05	0.8	10
B2	1.72	0.32	0.1	20	B2	1.09	0.13	0.1	20	B2	1.49	0.70	0.1	20
B2	1.82	0.17	0.2	20	B2	1.28	0.12	0.2	20	B2	1.69	0.29	0.2	20
B2	1.97	0.11	0.3	20	B2	1.59	0.09	0.3	20	B2	1.92	0.13	0.3	20
B2	2.03	0.08	0.4	20	B2	1.72	0.07	0.4	20	B2	1.96	0.10	0.4	20
B2	2.05	0.04	0.8	20	B2	1.99	0.04	0.8	20	B2	2.07	0.04	0.8	20
B2	1.96	0.32	0.1	40	B2	1.23	0.17	0.1	40	B2	1.85	0.64	0.1	40
B2	1.99	0.16	0.2	40	B2	1.64	0.13	0.2	40	B2	2.06	0.21	0.2	40
B2	2.02	0.11	0.3	40	B2	1.81	0.10	0.3	40	B2	2.04	0.13	0.3	40
B2	2.07	0.08	0.4	40	B2	1.98	0.08	0.4	40	B2	2.14	0.09	0.4	40
B2	1.99	0.04	0.8	40	B2	1.95	0.04	0.8	40	B2	1.99	0.04	0.8	40
B2	2.08	0.32	0.1	80	B2	1.49	0.22	0.1	80	B2	2.12	0.50	0.1	80
B2	2.35	0.17	0.2	80	B2	2.00	0.15	0.2	80	B2	2.33	0.19	0.2	80
B2	2.20	0.11	0.3	80	B2	2.06	0.10	0.3	80	B2	2.21	0.12	0.3	80
B2	2.04	0.08	0.4	80	B2	1.97	0.08	0.4	80	B2	2.05	0.09	0.4	80
B2	2.00	0.04	0.8	80	B2	1.98	0.04	0.8	80	B2	2.00	0.04	0.8	80
Ψ	0.99	0.05	0.1	5	Ψ	1.01	0.05	0.1	5	Ψ	1.02	0.05	0.1	5
Ψ	1.02	0.05	0.2	5	Ψ	1.05	0.05	0.2	5	Ψ	1.05	0.05	0.2	5
Ψ	0.92	0.05	0.3	5	Ψ	1.03	0.05	0.3	5	Ψ	1.03	0.05	0.3	5
Ψ	0.94	0.05	0.4	5	Ψ	1.01	0.05	0.4	5	Ψ	1.01	0.05	0.4	5
Ψ	1.02	0.05	0.8	5	Ψ	1.15	0.06	0.8	5	Ψ	1.15	0.06	0.8	5
Ψ	0.98	0.05	0.1	10	Ψ	1.01	0.05	0.1	10	Ψ	1.01	0.05	0.1	10
Ψ	0.99	0.05	0.2	10	Ψ	1.01	0.05	0.2	10	Ψ	1.01	0.05	0.2	10
Ψ	0.97	0.05	0.3	10	Ψ	1.04	0.05	0.3	10	Ψ	1.04	0.05	0.3	10
Ψ	1.04	0.05	0.4	10	Ψ	1.10	0.05	0.4	10	Ψ	1.10	0.05	0.4	10
Ψ	1.02	0.05	0.8	10	Ψ	1.14	0.06	0.8	10	Ψ	1.14	0.06	0.8	10
Ψ	1.05	0.05	0.1	20	Ψ	1.05	0.05	0.1	20	Ψ	1.05	0.05	0.1	20
Ψ	1.12	0.05	0.2	20	Ψ	1.12	0.05	0.2	20	Ψ	1.12	0.05	0.2	20
Ψ	0.98	0.05	0.3	20	Ψ	1.02	0.05	0.3	20	Ψ	1.02	0.05	0.3	20
Ψ	1.05	0.05	0.4	20	Ψ	1.11	0.05	0.4	20	Ψ	1.11	0.05	0.4	20

Ψ	1.01	0.05	0.8	20	Ψ	1.04	0.05	0.8	20	Ψ	1.04	0.05	0.8	20
Ψ	0.99	0.05	0.1	40	Ψ	1.00	0.05	0.1	40	Ψ	1.00	0.05	0.1	40
Ψ	1.00	0.05	0.2	40	Ψ	1.02	0.05	0.2	40	Ψ	1.02	0.05	0.2	40
Ψ	1.10	0.05	0.3	40	Ψ	1.11	0.05	0.3	40	Ψ	1.11	0.05	0.3	40
Ψ	0.98	0.04	0.4	40	Ψ	0.99	0.05	0.4	40	Ψ	0.99	0.05	0.4	40
Ψ	0.97	0.04	0.8	40	Ψ	0.97	0.04	0.8	40	Ψ	0.97	0.04	0.8	40
Ψ	1.01	0.05	0.1	80	Ψ	1.02	0.05	0.1	80	Ψ	1.02	0.05	0.1	80
Ψ	1.03	0.05	0.2	80	Ψ	1.05	0.05	0.2	80	Ψ	1.05	0.05	0.2	80
Ψ	1.02	0.05	0.3	80	Ψ	1.04	0.05	0.3	80	Ψ	1.04	0.05	0.3	80
Ψ	1.14	0.05	0.4	80	Ψ	1.15	0.05	0.4	80	Ψ	1.15	0.05	0.4	80
Ψ	1.00	0.05	0.8	80	Ψ	1.02	0.05	0.8	80	Ψ	1.02	0.05	0.8	80

4. Ordinal Simulations – Cumulative Probit Models

Table B.10. Ordinal, Ψ=0.2

Ψ=0.2	Model A					Model B					Model C			
	Estimate	SE	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj
B1	1.01	0.03	0.1	5	B1	1.01	0.03	0.1	5	B1	1.01	0.03	0.1	5
B1	0.99	0.03	0.2	5	B1	0.97	0.03	0.2	5	B1	0.97	0.03	0.2	5
B1	0.96	0.03	0.3	5	B1	0.96	0.03	0.3	5	B1	0.96	0.03	0.3	5
B1	0.98	0.03	0.4	5	B1	0.98	0.03	0.4	5	B1	0.98	0.03	0.4	5
B1	1.03	0.03	0.8	5	B1	0.81	0.03	0.8	5	B1	0.79	0.03	0.8	5
B1	1.03	0.02	0.1	10	B1	1.02	0.02	0.1	10	B1	1.02	0.02	0.1	10
B1	1.00	0.02	0.2	10	B1	1.00	0.02	0.2	10	B1	1.00	0.02	0.2	10
B1	1.01	0.02	0.3	10	B1	1.01	0.02	0.3	10	B1	1.01	0.02	0.3	10
B1	0.93	0.02	0.4	10	B1	0.96	0.02	0.4	10	B1	0.95	0.02	0.4	10
B1	0.78	0.02	0.8	10	B1	1.03	0.02	0.8	10	B1	1.03	0.02	0.8	10
B1	1.00	0.01	0.1	20	B1	1.01	0.01	0.1	20	B1	1.01	0.01	0.1	20
B1	1.00	0.01	0.2	20	B1	1.00	0.01	0.2	20	B1	1.00	0.01	0.2	20
B1	0.98	0.01	0.3	20	B1	0.98	0.01	0.3	20	B1	0.98	0.01	0.3	20

B1	0.99	0.01	0.4	20	B1	0.93	0.01	0.4	20	B1	0.92	0.01	0.4	20
B1	1.00	0.01	0.8	20	B1	0.75	0.01	0.8	20	B1	0.76	0.01	0.8	20
B1	1.01	0.01	0.1	40	B1	1.01	0.01	0.1	40	B1	1.01	0.01	0.1	40
B1	0.99	0.01	0.2	40	B1	0.99	0.01	0.2	40	B1	0.99	0.01	0.2	40
B1	0.97	0.01	0.3	40	B1	0.97	0.01	0.3	40	B1	0.97	0.01	0.3	40
B1	1.02	0.01	0.4	40	B1	1.02	0.01	0.4	40	B1	1.02	0.01	0.4	40
B1	0.69	0.01	0.8	40	B1	0.69	0.01	0.8	40	B1	0.69	0.01	0.8	40
B1	1.00	0.01	0.1	80	B1	1.00	0.01	0.1	80	B1	1.00	0.01	0.1	80
B1	0.99	0.01	0.2	80	B1	0.99	0.01	0.2	80	B1	0.99	0.01	0.2	80
B1	1.00	0.01	0.3	80	B1	1.00	0.01	0.3	80	B1	1.00	0.01	0.3	80
B1	1.00	0.01	0.4	80	B1	1.00	0.01	0.4	80	B1	1.00	0.01	0.4	80
B1	1.00	0.01	0.8	80	B1	1.00	0.01	0.8	80	B1	1.00	0.01	0.8	80
B2	2.03	0.24	0.1	5	B2	1.08	0.06	0.1	5	B2	1.87	0.72	0.1	5
B2	1.91	0.12	0.2	5	B2	1.24	0.05	0.2	5	B2	2.19	0.23	0.2	5
B2	1.99	0.09	0.3	5	B2	1.30	0.05	0.3	5	B2	2.15	0.18	0.3	5
B2	2.00	0.07	0.4	5	B2	1.41	0.05	0.4	5	B2	1.97	0.11	0.4	5
B2	2.00	0.05	0.8	5	B2	1.44	0.03	0.8	5	B2	1.69	0.05	0.8	5
B2	2.03	0.20	0.1	10	B2	1.24	0.06	0.1	10	B2	3.89	0.86	0.1	10
B2	1.83	0.10	0.2	10	B2	1.29	0.05	0.2	10	B2	1.93	0.17	0.2	10
B2	1.87	0.07	0.3	10	B2	1.38	0.05	0.3	10	B2	1.75	0.10	0.3	10
B2	1.72	0.05	0.4	10	B2	1.46	0.04	0.4	10	B2	1.83	0.07	0.4	10
B2	1.60	0.03	0.8	10	B2	1.84	0.04	0.8	10	B2	1.99	0.04	0.8	10
B2	1.92	0.17	0.1	20	B2	1.10	0.07	0.1	20	B2	2.00	0.82	0.1	20
B2	1.87	0.09	0.2	20	B2	1.36	0.06	0.2	20	B2	1.82	0.14	0.2	20
B2	2.00	0.06	0.3	20	B2	1.65	0.05	0.3	20	B2	2.01	0.07	0.3	20
B2	1.94	0.05	0.4	20	B2	1.52	0.04	0.4	20	B2	1.73	0.06	0.4	20
B2	2.02	0.03	0.8	20	B2	1.41	0.02	0.8	20	B2	1.46	0.03	0.8	20
B2	2.19	0.16	0.1	40	B2	1.31	0.09	0.1	40	B2	2.02	0.30	0.1	40
B2	2.12	0.08	0.2	40	B2	1.67	0.06	0.2	40	B2	2.07	0.10	0.2	40
B2	1.94	0.06	0.3	40	B2	1.75	0.05	0.3	40	B2	1.96	0.06	0.3	40
B2	2.06	0.04	0.4	40	B2	1.93	0.04	0.4	40	B2	2.07	0.05	0.4	40
B2	1.41	0.02	0.8	40	B2	1.39	0.02	0.8	40	B2	1.42	0.02	0.8	40
B2	1.96	0.15	0.1	80	B2	1.41	0.10	0.1	80	B2	1.92	0.23	0.1	80

B2	2.03	0.07	0.2	80	B2	1.79	0.08	0.2	80	B2	2.04	0.09	0.2	80
B2	2.08	0.05	0.3	80	B2	1.95	0.05	0.3	80	B2	2.08	0.06	0.3	80
B2	1.97	0.04	0.4	80	B2	1.89	0.04	0.4	80	B2	1.96	0.04	0.4	80
B2	1.98	0.02	0.8	80	B2	1.96	0.02	0.8	80	B2	1.98	0.02	0.8	80
L1	-1.02	0.03	0.1	5	L1	-1.03	0.03	0.1	5	L1	-1.02	0.03	0.1	5
L1	-0.92	0.03	0.2	5	L1	-0.93	0.03	0.2	5	L1	-0.93	0.03	0.2	5
L1	-0.94	0.03	0.3	5	L1	-0.95	0.03	0.3	5	L1	-0.97	0.03	0.3	5
L1	-1.02	0.03	0.4	5	L1	-1.03	0.03	0.4	5	L1	-1.01	0.03	0.4	5
L1	-1.00	0.03	0.8	5	L1	-0.82	0.03	0.8	5	L1	-0.86	0.03	0.8	5
L1	-1.02	0.02	0.1	10	L1	-1.02	0.02	0.1	10	L1	-0.98	0.03	0.1	10
L1	-0.99	0.02	0.2	10	L1	-0.99	0.02	0.2	10	L1	-0.99	0.02	0.2	10
L1	-1.01	0.02	0.3	10	L1	-1.02	0.03	0.3	10	L1	-1.03	0.03	0.3	10
L1	-0.85	0.02	0.4	10	L1	-0.92	0.02	0.4	10	L1	-0.86	0.02	0.4	10
L1	-0.72	0.02	0.8	10	L1	-1.02	0.03	0.8	10	L1	-1.01	0.03	0.8	10
L1	-1.01	0.02	0.1	20	L1	-1.00	0.02	0.1	20	L1	-1.01	0.02	0.1	20
L1	-0.97	0.02	0.2	20	L1	-0.97	0.02	0.2	20	L1	-0.98	0.02	0.2	20
L1	-0.98	0.02	0.3	20	L1	-0.97	0.02	0.3	20	L1	-0.97	0.02	0.3	20
L1	-1.00	0.02	0.4	20	L1	-0.95	0.02	0.4	20	L1	-0.94	0.02	0.4	20
L1	-1.04	0.03	0.8	20	L1	-0.83	0.02	0.8	20	L1	-0.83	0.02	0.8	20
L1	-1.02	0.02	0.1	40	L1	-1.02	0.02	0.1	40	L1	-1.02	0.02	0.1	40
L1	-0.98	0.02	0.2	40	L1	-0.98	0.02	0.2	40	L1	-0.99	0.02	0.2	40
L1	-0.97	0.02	0.3	40	L1	-0.99	0.02	0.3	40	L1	-0.96	0.02	0.3	40
L1	-1.02	0.02	0.4	40	L1	-1.02	0.02	0.4	40	L1	-1.02	0.02	0.4	40
L1	-0.74	0.01	0.8	40	L1	-0.74	0.02	0.8	40	L1	-0.74	0.02	0.8	40
L1	-0.99	0.02	0.1	80	L1	-0.98	0.02	0.1	80	L1	-0.99	0.02	0.1	80
L1	-1.03	0.02	0.2	80	L1	-1.03	0.02	0.2	80	L1	-1.03	0.02	0.2	80
L1	-0.99	0.02	0.3	80	L1	-0.99	0.02	0.3	80	L1	-0.99	0.02	0.3	80
L1	-0.99	0.02	0.4	80	L1	-0.99	0.02	0.4	80	L1	-0.99	0.02	0.4	80
L1	-0.99	0.02	0.8	80	L1	-0.98	0.02	0.8	80	L1	-0.98	0.02	0.8	80
L2	2.01	0.05	0.1	5	L2	2.01	0.05	0.1	5	L2	2.01	0.05	0.1	5
L2	2.05	0.05	0.2	5	L2	2.05	0.05	0.2	5	L2	2.05	0.05	0.2	5
L2	1.98	0.04	0.3	5	L2	1.97	0.05	0.3	5	L2	1.94	0.05	0.3	5
L2	1.98	0.05	0.4	5	L2	1.97	0.05	0.4	5	L2	1.98	0.05	0.4	5

L2	1.99	0.05	0.8	5	L2	1.63	0.04	0.8	5	L2	1.61	0.04	0.8	5
L2	2.01	0.03	0.1	10	L2	2.02	0.03	0.1	10	L2	2.05	0.04	0.1	10
L2	1.96	0.03	0.2	10	L2	1.96	0.03	0.2	10	L2	1.97	0.03	0.2	10
L2	2.01	0.03	0.3	10	L2	2.00	0.03	0.3	10	L2	1.99	0.03	0.3	10
L2	1.81	0.03	0.4	10	L2	1.85	0.03	0.4	10	L2	1.90	0.03	0.4	10
L2	1.61	0.03	0.8	10	L2	1.98	0.04	0.8	10	L2	1.98	0.04	0.8	10
L2	2.04	0.02	0.1	20	L2	2.05	0.03	0.1	20	L2	2.04	0.03	0.1	20
L2	2.01	0.02	0.2	20	L2	2.01	0.02	0.2	20	L2	2.00	0.02	0.2	20
L2	1.99	0.02	0.3	20	L2	1.99	0.02	0.3	20	L2	1.99	0.02	0.3	20
L2	2.01	0.02	0.4	20	L2	1.85	0.02	0.4	20	L2	1.86	0.02	0.4	20
L2	2.00	0.02	0.8	20	L2	1.55	0.02	0.8	20	L2	1.55	0.02	0.8	20
L2	1.99	0.02	0.1	40	L2	2.00	0.02	0.1	40	L2	2.00	0.02	0.1	40
L2	2.00	0.02	0.2	40	L2	2.00	0.02	0.2	40	L2	2.00	0.02	0.2	40
L2	1.94	0.02	0.2	40	L2	1.94	0.02	0.2	40	L2	1.97	0.02	0.2	40
L2	2.00	0.02	0.4	40	L2	2.00	0.02	0.4	40	L2	2.00	0.02	0.4	40
L2	1.48	0.02	0.8	40	L2	1.48	0.02	0.8	40	L2	1.48	0.02	0.8	40
L2	2.00	0.02	0.1	80	L2	2.00	0.02	0.1	80	L2	2.00	0.02	0.1	80
L2	1.98	0.02	0.2	80	L2	1.98	0.02	0.2	80	L2	1.98	0.02	0.2	80
L2	2.02	0.02	0.2	80	L2	2.02	0.02	0.2	80	L2	2.02	0.02	0.2	80
L2	2.00	0.02	0.4	80	L2	2.00	0.02	0.4	80	L2	2.00	0.02	0.4	80
L2	1.99	0.02	0.8	80	L2	2.00	0.02	0.8	80	L2	2.00	0.02	0.8	80
Ψ	0.23	0.03	0.1	5	Ψ	0.24	0.03	0.1	5	Ψ	0.24	0.03	0.1	5
Ψ	0.18	0.03	0.2	5	Ψ	0.20	0.03	0.2	5	Ψ	0.20	0.03	0.2	5
Ψ	0.21	0.03	0.3	5	Ψ	0.28	0.03	0.3	5	Ψ	0.28	0.03	0.3	5
Ψ	0.19	0.03	0.4	5	Ψ	0.30	0.04	0.4	5	Ψ	0.30	0.04	0.4	5
Ψ	0.15	0.03	0.8	5	Ψ	0.08	0.02	0.8	5	Ψ	0.18	0.03	0.8	5
Ψ	0.23	0.02	0.1	10	Ψ	0.23	0.02	0.1	10	Ψ	0.23	0.02	0.1	10
Ψ	0.20	0.02	0.2	10	Ψ	0.22	0.02	0.2	10	Ψ	0.22	0.02	0.2	10
Ψ	0.22	0.02	0.3	10	Ψ	0.26	0.02	0.3	10	Ψ	0.26	0.02	0.3	10
Ψ	0.23	0.03	0.4	10	Ψ	0.20	0.02	0.4	10	Ψ	0.24	0.02	0.4	10
Ψ	0.07	0.01	0.8	10	Ψ	0.30	0.03	0.8	10	Ψ	0.30	0.03	0.8	10
Ψ	0.21	0.01	0.1	20	Ψ	0.22	0.01	0.1	20	Ψ	0.22	0.01	0.1	20
Ψ	0.20	0.01	0.2	20	Ψ	0.22	0.01	0.2	20	Ψ	0.22	0.01	0.2	20

Ψ	0.18	0.01	0.3	20	Ψ	0.21	0.01	0.3	20	Ψ	0.21	0.01	0.3	20
Ψ	0.21	0.01	0.4	20	Ψ	0.24	0.02	0.4	20	Ψ	0.25	0.02	0.4	20
Ψ	0.22	0.02	0.8	20	Ψ	0.20	0.02	0.8	20	Ψ	0.20	0.02	0.8	20
Ψ	0.22	0.01	0.1	40	Ψ	0.23	0.01	0.1	40	Ψ	0.23	0.01	0.1	40
Ψ	0.20	0.01	0.2	40	Ψ	0.22	0.01	0.2	40	Ψ	0.22	0.01	0.2	40
Ψ	0.23	0.01	0.3	40	Ψ	0.25	0.01	0.3	40	Ψ	0.24	0.01	0.3	40
Ψ	0.20	0.01	0.4	40	Ψ	0.23	0.01	0.4	40	Ψ	0.23	0.01	0.4	40
Ψ	0.13	0.01	0.8	40	Ψ	0.15	0.01	0.8	40	Ψ	0.15	0.01	0.8	40
Ψ	0.20	0.01	0.1	80	Ψ	0.20	0.01	0.1	80	Ψ	0.20	0.01	0.1	80
Ψ	0.20	0.01	0.2	80	Ψ	0.21	0.01	0.2	80	Ψ	0.21	0.01	0.2	80
Ψ	0.20	0.01	0.3	80	Ψ	0.21	0.01	0.3	80	Ψ	0.21	0.01	0.3	80
Ψ	0.20	0.01	0.4	80	Ψ	0.22	0.01	0.4	80	Ψ	0.22	0.01	0.4	80
Ψ	0.19	0.01	0.8	80	Ψ	0.21	0.01	0.8	80	Ψ	0.21	0.01	0.8	80

Table B.11. Ordinal, Ψ=0.5

Ψ=0.5	Model A					Model B					Model C			
	Estimate	SE	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj
B1	0.96	0.03	0.1	5	B1	0.96	0.03	0.1	5	B1	0.96	0.03	0.1	5
B1	1.02	0.03	0.2	5	B1	1.04	0.03	0.2	5	B1	1.04	0.03	0.2	5
B1	0.99	0.03	0.3	5	B1	1.01	0.03	0.3	5	B1	1.01	0.03	0.3	5
B1	0.99	0.03	0.4	5	B1	1.00	0.03	0.4	5	B1	1.00	0.03	0.4	5
B1	1.03	0.03	0.8	5	B1	1.04	0.03	0.8	5	B1	1.04	0.03	0.8	5
B1	0.97	0.02	0.1	10	B1	0.98	0.02	0.1	10	B1	0.98	0.02	0.1	10
B1	1.01	0.02	0.2	10	B1	1.01	0.02	0.2	10	B1	1.01	0.02	0.2	10
B1	1.01	0.02	0.3	10	B1	1.01	0.02	0.3	10	B1	1.01	0.02	0.3	10
B1	1.01	0.02	0.4	10	B1	1.01	0.02	0.4	10	B1	1.01	0.02	0.4	10
B1	0.74	0.02	0.8	10	B1	0.75	0.02	0.8	10	B1	0.75	0.02	0.8	10
B1	0.99	0.01	0.1	20	B1	0.99	0.01	0.1	20	B1	0.99	0.01	0.1	20
B1	0.98	0.01	0.2	20	B1	0.98	0.01	0.2	20	B1	0.98	0.01	0.2	20
B1	0.98	0.01	0.3	20	B1	0.99	0.01	0.3	20	B1	0.99	0.01	0.3	20
B1	0.95	0.01	0.4	20	B1	0.95	0.01	0.4	20	B1	0.96	0.01	0.4	20
B1	1.01	0.01	0.8	20	B1	0.80	0.01	0.8	20	B1	0.80	0.01	0.8	20

B1	0.98	0.01	0.1	40	B1	0.99	0.01	0.1	40	B1	0.99	0.01	0.1	40
B1	0.96	0.01	0.2	40	B1	0.95	0.01	0.2	40	B1	0.96	0.01	0.2	40
B1	0.95	0.01	0.3	40	B1	0.95	0.01	0.3	40	B1	0.95	0.01	0.3	40
B1	0.89	0.01	0.4	40	B1	0.89	0.01	0.4	40	B1	0.89	0.01	0.4	40
B1	0.65	0.01	0.8	40	B1	0.66	0.01	0.8	40	B1	0.66	0.01	0.8	40
B1	0.96	0.01	0.1	80	B1	0.96	0.01	0.1	80	B1	0.96	0.00	0.1	80
B1	0.93	0.01	0.2	80	B1	0.93	0.01	0.2	80	B1	0.93	0.01	0.2	80
B1	0.92	0.01	0.3	80	B1	0.92	0.01	0.3	80	B1	0.92	0.01	0.3	80
B1	0.87	0.01	0.4	80	B1	0.86	0.01	0.4	80	B1	0.86	0.01	0.4	80
B1	1.01	0.01	0.8	80	B1	1.01	0.01	0.8	80	B1	1.01	0.01	0.8	80
B2	1.90	0.28	0.1	5	B2	0.99	0.07	0.1	5	B2	2.42	3.44	0.1	5
B2	1.97	0.15	0.2	5	B2	1.07	0.06	0.2	5	B2	1.24	0.33	0.2	5
B2	1.88	0.10	0.3	5	B2	1.27	0.06	0.3	5	B2	1.79	0.20	0.3	5
B2	1.89	0.08	0.4	5	B2	1.37	0.06	0.4	5	B2	1.82	0.12	0.4	5
B2	1.99	0.06	0.8	5	B2	1.74	0.05	0.8	5	B2	1.97	0.07	0.8	5
B2	1.86	0.26	0.1	10	B2	1.00	0.08	0.1	10	B2	1.14	0.64	0.1	10
B2	1.88	0.13	0.2	10	B2	1.22	0.07	0.2	10	B2	1.79	0.27	0.2	10
B2	1.98	0.09	0.3	10	B2	1.50	0.07	0.3	10	B2	2.18	0.15	0.3	10
B2	2.00	0.07	0.4	10	B2	1.60	0.06	0.4	10	B2	1.96	0.09	0.4	10
B2	1.45	0.03	0.8	10	B2	1.40	0.03	0.8	10	B2	1.50	0.04	0.8	10
B2	1.56	0.25	0.1	20	B2	1.19	0.10	0.1	20	B2	1.94	0.46	0.1	20
B2	2.10	0.13	0.2	20	B2	1.42	0.08	0.2	20	B2	1.92	0.18	0.2	20
B2	1.81	0.08	0.3	20	B2	1.56	0.07	0.3	20	B2	1.75	0.10	0.3	20
B2	1.74	0.06	0.4	20	B2	1.66	0.05	0.4	20	B2	1.88	0.07	0.4	20
B2	1.98	0.04	0.8	20	B2	1.47	0.03	0.8	20	B2	1.53	0.03	0.8	20
B2	1.90	0.23	0.1	40	B2	1.41	0.13	0.1	40	B2	2.39	0.42	0.1	40
B2	1.80	0.11	0.2	40	B2	1.55	0.08	0.2	40	B2	1.90	0.13	0.2	40
B2	1.82	0.08	0.3	40	B2	1.62	0.07	0.3	40	B2	1.81	0.09	0.3	40
B2	1.63	0.06	0.4	40	B2	1.52	0.05	0.4	40	B2	1.62	0.06	0.4	40
B2	1.44	0.03	0.8	40	B2	1.37	0.03	0.8	40	B2	1.39	0.03	0.8	40
B2	1.81	0.22	0.1	80	B2	1.27	0.15	0.1	80	B2	1.86	0.00	0.1	80
B2	1.85	0.10	0.2	80	B2	1.65	0.09	0.2	80	B2	1.93	0.12	0.2	80
B2	1.64	0.06	0.3	80	B2	1.52	0.06	0.3	80	B2	1.61	0.07	0.3	80

B2	1.63	0.05	0.4	80	B2	1.53	0.05	0.4	80	B2	1.58	0.05	0.4	80
B2	1.99	0.03	0.8	80	B2	1.98	0.03	0.8	80	B2	2.01	0.03	0.8	80
L1	-0.96	0.03	0.1	5	L1	-0.95	0.03	0.1	5	L1	-0.95	0.04	0.1	5
L1	-1.02	0.04	0.2	5	L1	-1.02	0.04	0.2	5	L1	-1.02	0.04	0.2	5
L1	-0.94	0.04	0.3	5	L1	-0.92	0.04	0.3	5	L1	-0.93	0.04	0.3	5
L1	-1.06	0.04	0.4	5	L1	-1.07	0.04	0.4	5	L1	-1.06	0.04	0.4	5
L1	-1.00	0.04	0.8	5	L1	-1.00	0.04	0.8	5	L1	-1.00	0.04	0.8	5
L1	-1.01	0.03	0.1	10	L1	-1.01	0.03	0.1	10	L1	-1.01	0.03	0.1	10
L1	-0.98	0.03	0.2	10	L1	-0.98	0.03	0.2	10	L1	-0.98	0.03	0.2	10
L1	-1.02	0.03	0.3	10	L1	-1.03	0.03	0.3	10	L1	-1.02	0.03	0.3	10
L1	-0.98	0.03	0.4	10	L1	-0.97	0.03	0.4	10	L1	-0.98	0.03	0.4	10
L1	-0.77	0.03	0.8	10	L1	-0.72	0.03	0.8	10	L1	-0.72	0.03	0.8	10
L1	-1.01	0.03	0.1	20	L1	-1.01	0.03	0.1	20	L1	-1.01	0.03	0.1	20
L1	-0.97	0.03	0.2	20	L1	-0.97	0.03	0.2	20	L1	-0.97	0.03	0.2	20
L1	-0.97	0.03	0.3	20	L1	-1.00	0.03	0.3	20	L1	-0.94	0.03	0.3	20
L1	-0.96	0.02	0.4	20	L1	-0.95	0.02	0.4	20	L1	-0.94	0.02	0.4	20
L1	-0.97	0.03	0.8	20	L1	-0.78	0.02	0.8	20	L1	-0.78	0.02	0.8	20
L1	-1.05	0.02	0.1	40	L1	-1.03	0.02	0.1	40	L1	-1.02	0.02	0.1	40
L1	-0.98	0.02	0.2	40	L1	-0.97	0.02	0.2	40	L1	-0.97	0.02	0.2	40
L1	-0.96	0.02	0.3	40	L1	-0.96	0.02	0.3	40	L1	-0.95	0.02	0.3	40
L1	-0.94	0.02	0.4	40	L1	-0.93	0.02	0.4	40	L1	-0.93	0.02	0.4	40
L1	-0.81	0.02	0.8	40	L1	-0.79	0.02	0.8	40	L1	-0.79	0.02	0.8	40
L1	-1.01	0.02	0.1	80	L1	-1.01	0.02	0.1	80	L1	-1.02	0.00	0.1	80
L1	-0.99	0.02	0.2	80	L1	-0.99	0.02	0.2	80	L1	-0.99	0.02	0.2	80
L1	-0.98	0.02	0.3	80	L1	-0.98	0.02	0.3	80	L1	-0.98	0.02	0.3	80
L1	-0.86	0.02	0.4	80	L1	-0.86	0.02	0.4	80	L1	-0.86	0.02	0.4	80
L1	-0.97	0.02	0.8	80	L1	-0.97	0.02	0.8	80	L1	-0.98	0.02	0.8	80
L2	1.93	0.05	0.1	5	L2	0.93	0.05	0.1	5	L2	1.94	0.05	0.1	5
L2	1.94	0.05	0.2	5	L2	1.93	0.05	0.2	5	L2	1.93	0.05	0.2	5
L2	1.93	0.05	0.3	5	L2	1.95	0.05	0.3	5	L2	1.95	0.05	0.3	5
L2	1.99	0.05	0.4	5	L2	1.98	0.05	0.4	5	L2	1.98	0.05	0.4	5
L2	2.07	0.06	0.8	5	L2	2.08	0.06	0.8	5	L2	2.08	0.06	0.8	5
L2	1.93	0.03	0.1	10	L2	1.93	0.04	0.1	10	L2	1.93	0.04	0.1	10

L2	2.04	0.04	0.2	10	L2	2.04	0.04	0.2	10	L2	2.04	0.04	0.2	10
L2	2.02	0.04	0.3	10	L2	2.01	0.04	0.3	10	L2	2.02	0.04	0.3	10
L2	2.05	0.04	0.4	10	L2	2.06	0.04	0.4	10	L2	2.04	0.04	0.4	10
L2	1.51	0.03	0.8	10	L2	1.49	0.03	0.8	10	L2	1.49	0.03	0.8	10
L2	2.01	0.03	0.1	20	L2	2.01	0.03	0.1	20	L2	2.00	0.03	0.1	20
L2	1.99	0.03	0.2	20	L2	1.99	0.03	0.2	20	L2	1.99	0.03	0.2	20
L2	1.96	0.03	0.3	20	L2	1.94	0.03	0.3	20	L2	2.01	0.03	0.3	20
L2	1.91	0.03	0.4	20	L2	1.93	0.03	0.4	20	L2	1.93	0.03	0.4	20
L2	2.02	0.03	0.8	20	L2	1.60	0.03	0.8	20	L2	1.60	0.03	0.8	20
L2	1.95	0.03	0.1	40	L2	1.95	0.03	0.1	40	L2	1.95	0.03	0.1	40
L2	1.83	0.02	0.2	40	L2	1.84	0.02	0.2	40	L2	1.84	0.02	0.2	40
L2	1.82	0.03	0.2	40	L2	1.82	0.03	0.2	40	L2	1.83	0.03	0.2	40
L2	1.81	0.02	0.4	40	L2	1.82	0.03	0.4	40	L2	1.82	0.03	0.4	40
L2	1.48	0.02	0.8	40	L2	1.44	0.02	0.8	40	L2	1.44	0.02	0.8	40
L2	1.90	0.02	0.1	80	L2	1.89	0.02	0.1	80	L2	1.89	0.00	0.1	80
L2	1.88	0.02	0.2	80	L2	1.88	0.02	0.2	80	L2	1.88	0.02	0.2	80
L2	1.86	0.02	0.2	80	L2	1.87	0.02	0.2	80	L2	1.87	0.02	0.2	80
L2	1.73	0.02	0.4	80	L2	1.72	0.02	0.4	80	L2	1.72	0.02	0.4	80
L2	2.01	0.03	0.8	80	L2	2.01	0.03	0.8	80	L2	2.02	0.03	0.8	80
Ψ	0.44	0.04	0.1	5	Ψ	0.45	0.04	0.1	5	Ψ	0.45	0.04	0.1	5
Ψ	0.46	0.05	0.2	5	Ψ	0.50	0.05	0.2	5	Ψ	0.50	0.05	0.2	5
Ψ	0.53	0.05	0.3	5	Ψ	0.57	0.05	0.3	5	Ψ	0.57	0.05	0.3	5
Ψ	0.46	0.05	0.4	5	Ψ	0.55	0.05	0.4	5	Ψ	0.55	0.05	0.4	5
Ψ	0.53	0.05	0.8	5	Ψ	0.70	0.06	0.8	5	Ψ	0.70	0.06	0.8	5
Ψ	0.44	0.03	0.1	10	Ψ	0.45	0.03	0.1	10	Ψ	0.45	0.03	0.1	10
Ψ	0.49	0.03	0.2	10	Ψ	0.52	0.04	0.2	10	Ψ	0.52	0.04	0.2	10
Ψ	0.48	0.03	0.3	10	Ψ	0.51	0.03	0.3	10	Ψ	0.51	0.04	0.3	10
Ψ	0.50	0.03	0.4	10	Ψ	0.56	0.04	0.4	10	Ψ	0.56	0.04	0.4	10
Ψ	0.32	0.03	0.8	10	Ψ	0.43	0.04	0.8	10	Ψ	0.43	0.04	0.8	10
Ψ	0.51	0.03	0.1	20	Ψ	0.51	0.03	0.1	20	Ψ	0.51	0.03	0.1	20
Ψ	0.54	0.03	0.2	20	Ψ	0.57	0.03	0.2	20	Ψ	0.57	0.03	0.2	20
Ψ	0.49	0.03	0.3	20	Ψ	0.53	0.03	0.3	20	Ψ	0.51	0.03	0.3	20
Ψ	0.40	0.02	0.4	20	Ψ	0.43	0.02	0.4	20	Ψ	0.43	0.02	0.4	20

Ψ	0.44	0.03	0.8	20	Ψ	0.36	0.02	0.8	20	Ψ	0.36	0.02	0.8	20
Ψ	0.47	0.02	0.1	40	Ψ	0.51	0.03	0.1	40	Ψ	0.51	0.03	0.1	40
Ψ	0.42	0.02	0.2	40	Ψ	0.43	0.02	0.2	40	Ψ	0.43	0.02	0.2	40
Ψ	0.47	0.02	0.3	40	Ψ	0.49	0.02	0.3	40	Ψ	0.49	0.02	0.3	40
Ψ	0.45	0.02	0.4	40	Ψ	0.46	0.02	0.4	40	Ψ	0.46	0.02	0.4	40
Ψ	0.34	0.02	0.8	40	Ψ	0.31	0.02	0.8	40	Ψ	0.32	0.02	0.8	40
Ψ	0.44	0.02	0.1	80	Ψ	0.46	0.02	0.1	80	Ψ	0.44	.	0.1	80
Ψ	0.36	0.01	0.2	80	Ψ	0.38	0.02	0.2	80	Ψ	0.38	0.02	0.2	80
Ψ	0.35	0.01	0.3	80	Ψ	0.37	0.02	0.3	80	Ψ	0.37	0.02	0.3	80
Ψ	0.36	0.02	0.4	80	Ψ	0.36	0.02	0.4	80	Ψ	0.36	0.02	0.4	80
Ψ	0.52	0.03	0.8	80	Ψ	0.52	0.03	0.8	80	Ψ	0.53	0.03	0.8	80

Table B.12. Ordinal, Ψ=1.0

Ψ=1.0	Model A					Model B					Model C			
	Estimate	SE	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj
B1	1.02	0.03	0.1	5	B1	1.02	0.03	0.1	5	B1	1.02	0.03	0.1	5
B1	1.03	0.03	0.2	5	B1	1.03	0.03	0.2	5	B1	0.99	0.03	0.2	5
B1	0.97	0.03	0.3	5	B1	0.98	0.03	0.3	5	B1	0.98	0.03	0.3	5
B1	0.97	0.03	0.4	5	B1	0.97	0.03	0.4	5	B1	0.97	0.03	0.4	5
B1	0.74	0.02	0.8	5	B1	0.79	0.03	0.8	5	B1	0.76	0.03	0.8	5
B1	1.00	0.02	0.1	10	B1	1.00	0.02	0.1	10	B1	1.00	0.02	0.1	10
B1	1.05	0.02	0.2	10	B1	1.04	0.02	0.2	10	B1	1.05	0.02	0.2	10
B1	0.97	0.02	0.3	10	B1	0.96	0.02	0.3	10	B1	0.96	0.02	0.3	10
B1	0.90	0.02	0.4	10	B1	0.91	0.02	0.4	10	B1	0.91	0.02	0.4	10
B1	0.74	0.02	0.8	10	B1	0.73	0.02	0.8	10	B1	0.73	0.02	0.8	10
B1	0.96	0.01	0.1	20	B1	0.96	0.01	0.1	20	B1	0.96	0.01	0.1	20
B1	0.91	0.01	0.2	20	B1	0.91	0.01	0.2	20	B1	0.91	0.01	0.2	20
B1	1.01	0.01	0.3	20	B1	1.02	0.01	0.3	20	B1	1.02	0.01	0.3	20
B1	0.89	0.01	0.4	20	B1	0.91	0.01	0.4	20	B1	0.88	0.01	0.4	20
B1	0.78	0.01	0.8	20	B1	0.72	0.01	0.8	20	B1	0.76	0.01	0.8	20
B1	0.93	0.01	0.1	40	B1	0.94	0.01	0.1	40	B1	0.94	0.01	0.1	40
B1	0.83	0.01	0.2	40	B1	0.83	0.01	0.2	40	B1	0.83	0.01	0.2	40

B1	0.85	0.01	0.3	40	B1	0.85	0.01	0.3	40	B1	0.85	0.01	0.3	40
B1	0.92	0.01	0.4	40	B1	0.90	0.01	0.4	40	B1	0.90	0.01	0.4	40
B1	0.73	0.01	0.8	40	B1	0.72	0.01	0.8	40	B1	0.71	0.01	0.8	40
B1	0.84	0.01	0.1	80	B1	0.84	0.01	0.1	80	B1	0.84	0.01	0.1	80
B1	0.82	0.01	0.2	80	B1	0.82	0.01	0.2	80	B1	0.82	0.01	0.2	80
B1	0.84	0.01	0.3	80	B1	0.84	0.01	0.3	80	B1	0.84	0.01	0.3	80
B1	0.82	0.01	0.4	80	B1	0.82	0.01	0.4	80	B1	0.82	0.01	0.4	80
B1	1.01	0.01	0.8	80	B1	1.01	0.01	0.8	80	B1	1.01	0.01	0.8	80
B2	2.69	0.37	0.1	5	B2	1.08	0.09	0.1	5	B2	2.01	1.37	0.1	5
B2	2.00	0.20	0.2	5	B2	1.20	0.08	0.2	5	B2	2.19	0.45	0.2	5
B2	1.96	0.13	0.3	5	B2	1.22	0.08	0.3	5	B2	1.78	0.25	0.3	5
B2	1.74	0.09	0.4	5	B2	1.37	0.07	0.4	5	B2	1.77	0.15	0.4	5
B2	1.42	0.05	0.8	5	B2	1.28	0.04	0.8	5	B2	1.53	0.06	0.8	5
B2	1.76	0.37	0.1	10	B2	1.05	0.10	0.1	10	B2	1.31	0.75	0.1	10
B2	1.95	0.17	0.2	10	B2	1.42	0.10	0.2	10	B2	2.07	0.31	0.2	10
B2	1.89	0.12	0.3	10	B2	1.36	0.09	0.3	10	B2	1.71	0.19	0.3	10
B2	1.51	0.07	0.4	10	B2	1.25	0.06	0.4	10	B2	1.49	0.09	0.4	10
B2	1.46	0.04	0.8	10	B2	1.34	0.04	0.8	10	B2	1.45	0.04	0.8	10
B2	1.32	0.30	0.1	20	B2	0.98	0.13	0.1	20	B2	1.23	0.67	0.1	20
B2	1.32	0.14	0.2	20	B2	0.93	0.10	0.2	20	B2	1.25	0.24	0.2	20
B2	1.54	0.10	0.3	20	B2	1.22	0.08	0.3	20	B2	1.44	0.12	0.3	20
B2	1.45	0.07	0.4	20	B2	1.23	0.06	0.4	20	B2	1.40	0.09	0.4	20
B2	1.45	0.04	0.8	20	B2	1.49	0.04	0.8	20	B2	1.52	0.04	0.8	20
B2	1.58	0.30	0.1	40	B2	0.92	0.16	0.1	40	B2	1.31	0.61	0.1	40
B2	1.41	0.12	0.2	40	B2	1.16	0.10	0.2	40	B2	1.44	0.16	0.2	40
B2	1.44	0.11	0.3	40	B2	1.29	0.10	0.3	40	B2	1.44	0.12	0.3	40
B2	1.62	0.07	0.4	40	B2	1.53	0.07	0.4	40	B2	1.63	0.09	0.4	40
B2	1.45	0.03	0.8	40	B2	1.49	0.03	0.8	40	B2	1.52	0.03	0.8	40
B2	1.42	0.25	0.1	80	B2	1.01	0.17	0.1	80	B2	1.39	0.39	0.1	80
B2	1.65	0.12	0.2	80	B2	1.40	0.11	0.2	80	B2	1.62	0.14	0.2	80
B2	1.69	0.11	0.3	80	B2	1.58	0.10	0.3	80	B2	1.68	0.12	0.3	80
B2	1.60	0.08	0.4	80	B2	1.56	0.08	0.4	80	B2	1.60	0.09	0.4	80
B2	2.01	0.04	0.8	80	B2	1.98	0.04	0.8	80	B2	2.00	0.04	0.8	80

L1	-0.98	0.04	0.1	5	L1	-0.98	0.04	0.1	5	L1	-0.99	0.04	0.1	5
L1	-1.00	0.05	0.2	5	L1	-1.00	0.05	0.2	5	L1	-0.97	0.05	0.2	5
L1	-1.08	0.05	0.3	5	L1	-1.08	0.05	0.3	5	L1	-1.08	0.05	0.3	5
L1	-0.99	0.04	0.4	5	L1	-0.97	0.04	0.4	5	L1	-0.97	0.04	0.4	5
L1	-0.75	0.04	0.8	5	L1	-0.93	0.04	0.8	5	L1	-0.79	0.04	0.8	5
L1	-1.00	0.04	0.1	10	L1	-1.00	0.04	0.1	10	L1	-1.00	0.04	0.1	10
L1	-1.05	0.04	0.2	10	L1	1.03	0.04	0.2	10	L1	-1.03	0.04	0.2	10
L1	-1.03	0.04	0.3	10	L1	-0.95	0.04	0.3	10	L1	-0.96	0.04	0.3	10
L1	-1.03	0.03	0.4	10	L1	-1.03	0.03	0.4	10	L1	-1.03	0.03	0.4	10
L1	-0.83	0.03	0.8	10	L1	-0.83	0.03	0.8	10	L1	-0.82	0.03	0.8	10
L1	-0.95	0.03	0.1	20	L1	-0.94	0.03	0.1	20	L1	-0.95	0.03	0.1	20
L1	-1.00	0.03	0.2	20	L1	-1.01	0.03	0.2	20	L1	-1.01	0.03	0.2	20
L1	0.99	0.03	0.3	20	L1	-0.99	0.03	0.3	20	L1	-0.99	0.03	0.3	20
L1	-1.01	0.03	0.4	20	L1	-1.02	0.03	0.4	20	L1	-1.02	0.03	0.4	20
L1	-0.82	0.03	0.8	20	L1	-0.82	0.03	0.8	20	L1	-0.82	0.03	0.8	20
L1	-0.95	0.03	0.1	40	L1	-0.95	0.03	0.1	40	L1	-0.95	0.03	0.1	40
L1	-0.99	0.03	0.2	40	L1	-0.99	0.03	0.2	40	L1	-0.99	0.03	0.2	40
L1	-0.93	0.03	0.3	40	L1	-0.93	0.03	0.3	40	L1	-0.93	0.03	0.3	40
L1	-0.98	0.03	0.4	40	L1	-0.95	0.03	0.4	40	L1	-0.95	0.03	0.4	40
L1	-0.75	0.03	0.8	40	L1	-0.77	0.03	0.8	40	L1	-0.77	0.03	0.8	40
L1	-0.82	0.03	0.1	80	L1	-0.81	0.03	0.1	80	L1	-0.82	0.03	0.1	80
L1	-0.83	0.03	0.2	80	L1	-0.83	0.02	0.2	80	L1	-0.83	0.02	0.2	80
L1	-0.77	0.03	0.3	80	L1	-0.76	0.03	0.3	80	L1	-0.76	0.03	0.3	80
L1	-0.73	0.03	0.4	80	L1	-0.73	0.03	0.4	80	L1	-0.73	0.03	0.4	80
L1	-0.97	0.03	0.8	80	L1	-0.97	0.03	0.8	80	L1	-0.97	0.03	0.8	80
L2	2.03	0.05	0.1	5	L2	2.03	0.06	0.1	5	L2	2.02	0.06	0.1	5
L2	2.07	0.06	0.2	5	L2	2.07	0.06	0.2	5	L2	2.03	0.06	0.2	5
L2	1.91	0.05	0.3	5	L2	1.91	0.06	0.3	5	L2	1.91	0.06	0.3	5
L2	1.88	0.05	0.4	5	L2	1.96	0.05	0.4	5	L2	1.95	0.05	0.4	5
L2	1.45	0.04	0.8	5	L2	1.63	0.05	0.8	5	L2	1.51	0.05	0.8	5
L2	2.02	0.04	0.1	10	L2	2.02	0.04	0.1	10	L2	2.02	0.04	0.1	10
L2	2.01	0.04	0.2	10	L2	2.01	0.04	0.2	10	L2	2.02	0.05	0.2	10
L2	1.89	0.04	0.3	10	L2	2.02	0.05	0.3	10	L2	2.02	0.05	0.3	10

L2	1.74	0.04	0.4	10	L2	1.72	0.04	0.4	10	L2	1.72	0.04	0.4	10
L2	1.41	0.03	0.8	10	L2	1.42	0.04	0.8	10	L2	1.41	0.03	0.8	10
L2	1.97	0.04	0.1	20	L2	1.97	0.04	0.1	20	L2	1.97	0.04	0.1	20
L2	1.74	0.03	0.2	20	L2	1.74	0.03	0.2	20	L2	1.73	0.03	0.2	20
L2	1.81	0.03	0.3	20	L2	1.82	0.03	0.3	20	L2	1.81	0.03	0.3	20
L2	1.66	0.03	0.4	20	L2	1.66	0.03	0.4	20	L2	1.68	0.03	0.4	20
L2	1.51	0.03	0.8	20	L2	1.57	0.03	0.8	20	L2	1.55	0.03	0.8	20
L2	1.80	0.03	0.1	40	L2	1.80	0.03	0.1	40	L2	1.80	0.03	0.1	40
L2	1.66	0.03	0.2	40	L2	1.67	0.03	0.2	40	L2	1.66	0.03	0.2	40
L2	1.75	0.03	0.2	40	L2	1.75	0.03	0.2	40	L2	1.75	0.03	0.2	40
L2	1.72	0.03	0.4	40	L2	1.73	0.03	0.4	40	L2	1.72	0.03	0.4	40
L2	1.39	0.03	0.8	40	L2	1.44	0.03	0.8	40	L2	1.45	0.03	0.8	40
L2	1.57	0.03	0.1	80	L2	1.57	0.03	0.1	80	L2	1.57	0.03	0.1	80
L2	1.55	0.03	0.2	80	L2	1.55	0.02	0.2	80	L2	1.55	0.02	0.2	80
L2	1.65	0.03	0.2	80	L2	1.66	0.03	0.2	80	L2	1.65	0.03	0.2	80
L2	1.67	0.03	0.4	80	L2	1.68	0.03	0.4	80	L2	1.68	0.03	0.4	80
L2	2.05	0.03	0.8	80	L2	2.05	0.03	0.8	80	L2	2.05	0.03	0.8	80
Ψ	0.99	0.08	0.1	5	Ψ	1.02	0.08	0.1	5	Ψ	1.02	0.08	0.1	5
Ψ	1.14	0.09	0.2	5	Ψ	1.17	0.09	0.2	5	Ψ	1.14	0.09	0.2	5
Ψ	1.10	0.08	0.3	5	Ψ	1.17	0.09	0.3	5	Ψ	1.17	0.09	0.3	5
Ψ	0.91	0.07	0.4	5	Ψ	0.98	0.08	0.4	5	Ψ	1.00	0.08	0.4	5
Ψ	0.72	0.07	0.8	5	Ψ	0.69	0.05	0.8	5	Ψ	0.86	0.08	0.8	5
Ψ	1.04	0.06	0.1	10	Ψ	1.05	0.06	0.1	10	Ψ	1.05	0.06	0.1	10
Ψ	1.06	0.06	0.2	10	Ψ	1.09	0.07	0.2	10	Ψ	1.08	0.07	0.2	10
Ψ	0.99	0.06	0.3	10	Ψ	1.19	0.08	0.3	10	Ψ	1.18	0.08	0.3	10
Ψ	0.63	0.03	0.4	10	Ψ	0.65	0.03	0.4	10	Ψ	0.64	0.03	0.4	10
Ψ	0.62	0.04	0.8	10	Ψ	0.64	0.04	0.8	10	Ψ	0.64	0.04	0.8	10
Ψ	0.89	0.04	0.1	20	Ψ	0.88	0.04	0.1	20	Ψ	0.89	0.04	0.1	20
Ψ	0.70	0.03	0.2	20	Ψ	0.70	0.03	0.2	20	Ψ	0.68	0.03	0.2	20
Ψ	0.74	0.03	0.3	20	Ψ	0.75	0.03	0.3	20	Ψ	0.74	0.03	0.3	20
Ψ	0.63	0.03	0.4	20	Ψ	0.65	0.03	0.4	20	Ψ	0.71	0.03	0.4	20
Ψ	0.58	0.03	0.8	20	Ψ	0.64	0.03	0.8	20	Ψ	0.62	0.03	0.8	20
Ψ	0.87	0.04	0.1	40	Ψ	0.87	0.04	0.1	40	Ψ	0.87	0.04	0.1	40

Ψ	0.54	0.02	0.2	40	Ψ	0.54	0.02	0.2	40	Ψ	0.54	0.02	0.2	40
Ψ	0.91	0.04	0.3	40	Ψ	0.92	0.04	0.3	40	Ψ	0.92	0.04	0.3	40
Ψ	0.69	0.03	0.4	40	Ψ	0.90	0.05	0.4	40	Ψ	0.90	0.05	0.4	40
Ψ	0.54	0.03	0.8	40	Ψ	0.54	0.02	0.8	40	Ψ	0.54	0.02	0.8	40
Ψ	0.60	0.02	0.1	80	Ψ	0.61	0.03	0.1	80	Ψ	0.61	0.03	0.1	80
Ψ	0.55	0.02	0.2	80	Ψ	0.55	0.02	0.2	80	Ψ	0.55	0.02	0.2	80
Ψ	1.00	0.07	0.3	80	Ψ	1.01	0.07	0.3	80	Ψ	1.01	0.07	0.3	80
Ψ	1.10	0.07	0.4	80	Ψ	1.11	0.07	0.4	80	Ψ	1.10	0.07	0.4	80
Ψ	0.98	0.05	0.8	80	Ψ	1.01	0.05	0.8	80	Ψ	1.01	0.05	0.8	80

5. Nominal Logistic Regression Simulation

Table B.13. Nominal, Ψ=1.0, 1.0

Ψ1=1.0	Model A				Model B					Model C				
Ψ2=1.0	Estimate	SE	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj
B11	1.02	0.07	0.1	5	B11	1.06	0.08	0.1	5	B11	1.05	0.08	0.1	5
B11	1.11	0.07	0.4	5	B11	1.09	0.08	0.4	5	B11	1.09	0.08	0.4	5
B11	0.96	0.07	0.8	5	B11	0.88	0.07	0.8	5	B11	0.88	0.07	0.8	5
B11	0.97	0.05	0.1	10	B11	0.96	0.05	0.1	10	B11	0.96	0.05	0.1	10
B11	1.11	0.05	0.4	10	B11	1.10	0.05	0.4	10	B11	1.10	0.05	0.4	10
B11	0.99	0.05	0.8	10	B11	0.96	0.05	0.8	10	B11	0.96	0.05	0.8	10
B11	0.97	0.03	0.1	20	B11	0.97	0.03	0.1	20	B11	0.97	0.03	0.1	20
B11	1.00	0.04	0.4	20	B11	0.99	0.04	0.4	20	B11	0.99	0.04	0.4	20
B11	1.00	0.03	0.8	20	B11	0.99	0.03	0.8	20	B11	0.99	0.03	0.8	20
B12	3.09	0.10	0.1	5	B12	3.13	0.11	0.1	5	B12	3.13	0.11	0.1	5
B12	3.28	0.11	0.4	5	B12	3.27	0.11	0.4	5	B12	3.27	0.11	0.4	5
B12	3.27	0.12	0.8	5	B12	3.22	0.12	0.8	5	B12	3.22	0.12	0.8	5
B12	2.94	0.07	0.1	10	B12	2.94	0.07	0.1	10	B12	2.94	0.07	0.1	10
B12	3.05	0.07	0.4	10	B12	3.04	0.07	0.4	10	B12	3.04	0.07	0.4	10
B12	3.16	0.08	0.8	10	B12	3.18	0.08	0.8	10	B12	3.18	0.08	0.8	10
B12	2.98	0.05	0.1	20	B12	2.97	0.05	0.1	20	B12	2.97	0.05	0.1	20
B12	3.02	0.05	0.4	20	B12	3.02	0.05	0.4	20	B12	3.02	0.05	0.4	20

B12	3.00	0.05	0.8	20	B12	2.99	0.05	0.8	20	B12	2.99	0.05	0.8	20
B21	2.10	0.61	0.1	5	B21	0.86	0.14	0.1	5	B21	-1.35	1.83	0.1	5
B21	1.89	0.17	0.4	5	B21	1.40	0.12	0.4	5	B21	1.80	0.26	0.4	5
B21	1.93	0.12	0.8	5	B21	1.67	0.10	0.8	5	B21	1.94	0.13	0.8	5
B21	2.68	0.48	0.1	10	B21	1.23	0.15	0.1	10	B21	4.53	2.04	0.1	10
B21	2.04	0.14	0.4	10	B21	1.55	0.11	0.4	10	B21	1.83	0.18	0.4	10
B21	2.06	0.09	0.8	10	B21	1.80	0.09	0.8	10	B21	1.93	0.10	0.8	10
B21	1.29	0.39	0.1	20	B21	1.11	0.17	0.1	20	B21	2.56	1.88	0.1	20
B21	2.03	0.12	0.4	20	B21	1.69	0.11	0.4	20	B21	1.91	0.14	0.4	20
B21	2.00	0.07	0.8	20	B21	1.89	0.07	0.8	20	B21	1.96	0.07	0.8	20
B22	4.53	0.63	0.1	5	B22	3.03	0.16	0.1	5	B22	1.92	1.87	0.1	5
B22	4.91	0.22	0.4	5	B22	3.90	0.16	0.4	5	B22	4.74	0.31	0.4	5
B22	5.33	0.19	0.8	5	B22	4.82	0.18	0.8	5	B22	5.37	0.21	0.8	5
B22	4.91	0.50	0.1	10	B22	3.09	0.16	0.1	10	B22	4.95	2.13	0.1	10
B22	4.97	0.16	0.4	10	B22	4.12	0.13	0.4	10	B22	4.79	0.19	0.4	10
B22	5.18	0.13	0.8	10	B22	4.80	0.12	0.8	10	B22	5.07	0.13	0.8	10
B22	5.14	0.41	0.1	20	B22	3.38	0.18	0.1	20	B22	7.48	1.98	0.1	20
B22	5.00	0.13	0.4	20	B22	4.43	0.11	0.4	20	B22	4.88	0.14	0.4	20
B22	4.92	0.09	0.8	20	B22	4.76	0.09	0.8	20	B22	4.89	0.09	0.8	20
Int1	1.01	0.08	0.1	5	Int1	1.00	0.08	0.1	5	Int1	1.01	0.08	0.1	5
Int1	1.13	0.09	0.4	5	Int1	1.09	0.09	0.4	5	Int1	1.08	0.09	0.4	5
Int1	0.81	0.10	0.8	5	Int1	0.78	0.09	0.8	5	Int1	0.78	0.09	0.8	5
Int1	1.05	0.06	0.1	10	Int1	1.03	0.06	0.1	10	Int1	0.99	0.07	0.1	10
Int1	1.08	0.07	0.4	10	Int1	1.04	0.07	0.4	10	Int1	1.04	0.07	0.4	10
Int1	1.06	0.07	0.8	10	Int1	0.98	0.07	0.8	10	Int1	0.98	0.07	0.8	10
Int1	0.96	0.05	0.1	20	Int1	0.96	0.05	0.1	20	Int1	0.98	0.05	0.1	20
Int1	0.96	0.05	0.4	20	Int1	0.95	0.05	0.4	20	Int1	0.94	0.05	0.4	20
Int1	1.06	0.06	0.8	20	Int1	1.03	0.05	0.8	20	Int1	1.04	0.05	0.8	20
Int2	2.00	0.08	0.1	5	Int2	2.00	0.08	0.1	5	Int2	2.00	0.08	0.1	5
Int2	2.13	0.09	0.4	5	Int2	2.11	0.09	0.4	5	Int2	2.09	0.09	0.4	5
Int2	1.95	0.09	0.8	5	Int2	1.95	0.09	0.8	5	Int2	1.96	0.09	0.8	5
Int2	2.08	0.06	0.1	10	Int2	2.07	0.06	0.1	10	Int2	2.04	0.07	0.1	10
Int2	2.13	0.06	0.4	10	Int2	2.10	0.06	0.4	10	Int2	2.09	0.06	0.4	10

Int2	2.08	0.07	0.8	10	Int2	2.02	0.07	0.8	10	Int2	2.01	0.07	0.8	10
Int2	1.98	0.05	0.1	20	Int2	1.97	0.05	0.1	20	Int2	2.01	0.05	0.1	20
Int2	2.05	0.05	0.4	20	Int2	2.04	0.05	0.4	20	Int2	2.04	0.05	0.4	20
Int2	2.00	0.05	0.8	20	Int2	1.97	0.06	0.8	20	Int2	1.97	0.06	0.8	20
Ψ1	1.10	0.17	0.1	5	Ψ1	1.10	0.17	0.1	5	Ψ1	1.10	0.17	0.1	5
Ψ1	0.90	0.16	0.4	5	Ψ1	0.89	0.16	0.4	5	Ψ1	0.89	0.16	0.4	5
Ψ1	1.33	0.20	0.8	5	Ψ1	1.28	0.20	0.8	5	Ψ1	1.28	0.20	0.8	5
Ψ1	0.88	0.10	0.1	10	Ψ1	0.88	0.10	0.1	10	Ψ1	0.88	0.10	0.1	10
Ψ1	1.04	0.11	0.4	10	Ψ1	1.04	0.11	0.4	10	Ψ1	1.04	0.11	0.4	10
Ψ1	1.15	0.12	0.8	10	Ψ1	1.17	0.13	0.8	10	Ψ1	1.17	0.13	0.8	10
Ψ1	0.94	0.08	0.1	20	Ψ1	0.94	0.08	0.1	20	Ψ1	0.94	0.08	0.1	20
Ψ1	1.14	0.09	0.4	20	Ψ1	1.15	0.09	0.4	20	Ψ1	1.15	0.09	0.4	20
Ψ1	0.96	0.08	0.8	20	Ψ1	0.98	0.08	0.8	20	Ψ1	0.98	0.08	0.8	20
Ψ2	1.04	0.19	0.1	5	Ψ2	1.05	0.19	0.1	5	Ψ2	1.05	0.19	0.1	5
Ψ2	1.33	0.22	0.4	5	Ψ2	1.51	0.24	0.4	5	Ψ2	1.51	0.24	0.4	5
Ψ2	1.30	0.26	0.8	5	Ψ2	1.73	0.30	0.8	5	Ψ2	1.73	0.30	0.8	5
Ψ2	1.02	0.11	0.1	10	Ψ2	1.04	0.11	0.1	10	Ψ2	1.04	0.11	0.1	10
Ψ2	0.86	0.11	0.4	10	Ψ2	1.04	0.12	0.4	10	Ψ2	1.04	0.12	0.4	10
Ψ2	1.06	0.14	0.8	10	Ψ2	1.42	0.16	0.8	10	Ψ2	1.42	0.16	0.8	10
Ψ2	1.04	0.08	0.1	20	Ψ2	1.08	0.08	0.1	20	Ψ2	1.08	0.08	0.1	20
Ψ2	0.98	0.08	0.4	20	Ψ2	1.12	0.09	0.4	20	Ψ2	1.12	0.09	0.4	20
Ψ2	0.99	0.09	0.8	20	Ψ2	1.15	0.10	0.8	20	Ψ2	1.15	0.10	0.8	20

Table B.14. Nominal, Ψ=0.5, 1.0

Ψ1=0.5	Model A					Model B					Model C			
Ψ2=1.0	Estimate	SE	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj		Estimate	S.e.	τ_{xx}	Nj
B11	0.93	0.07	0.1	5	B11	0.94	0.07	0.1	5	B11	0.94	0.07	0.1	5
B11	0.97	0.07	0.4	5	B11	0.91	0.07	0.4	5	B11	0.91	0.07	0.4	5
B11	0.96	0.07	0.8	5	B11	0.86	0.07	0.8	5	B11	0.86	0.07	0.8	5
B11	0.99	0.05	0.1	10	B11	0.97	0.05	0.1	10	B11	0.97	0.05	0.1	10
B11	1.08	0.05	0.4	10	B11	1.04	0.05	0.4	10	B11	1.04	0.05	0.4	10
B11	1.02	0.05	0.8	10	B11	0.98	0.05	0.8	10	B11	0.98	0.05	0.8	10
B11	1.05	0.03	0.1	20	B11	1.04	0.03	0.1	20	B11	1.04	0.03	0.1	20
B11	1.00	0.03	0.4	20	B11	0.98	0.03	0.4	20	B11	0.98	0.03	0.4	20
B11	1.04	0.03	0.8	20	B11	1.02	0.03	0.8	20	B11	1.02	0.03	0.8	20
B12	3.07	0.10	0.1	5	B12	3.07	0.10	0.1	5	B12	3.07	0.10	0.1	5
B12	3.02	0.10	0.4	5	B12	2.95	0.10	0.4	5	B12	2.95	0.10	0.4	5
B12	2.91	0.10	0.8	5	B12	2.86	0.11	0.8	5	B12	2.86	0.11	0.8	5
B12	3.01	0.07	0.1	10	B12	2.99	0.07	0.1	10	B12	2.99	0.07	0.1	10
B12	3.13	0.07	0.4	10	B12	3.10	0.07	0.4	10	B12	3.10	0.07	0.4	10
B12	2.93	0.07	0.8	10	B12	2.90	0.07	0.8	10	B12	2.90	0.07	0.8	10
B12	3.02	0.05	0.1	20	B12	3.01	0.05	0.1	20	B12	3.01	0.05	0.1	20
B12	3.08	0.05	0.4	20	B12	3.07	0.05	0.4	20	B12	3.07	0.05	0.4	20
B12	3.01	0.05	0.8	20	B12	3.00	0.05	0.8	20	B12	3.00	0.05	0.8	20
B21	1.27	0.47	0.1	5	B21	0.87	0.12	0.1	5	B21	0.01	1.53	0.1	5
B21	2.06	0.15	0.4	5	B21	1.46	0.11	0.4	5	B21	2.20	0.23	0.4	5
B21	1.87	0.10	0.8	5	B21	1.62	0.09	0.8	5	B21	1.87	0.11	0.8	5
B21	2.30	0.37	0.1	10	B21	1.26	0.12	0.1	10	B21	4.90	1.63	0.1	10
B21	2.07	0.12	0.4	10	B21	1.66	0.09	0.4	10	B21	2.05	0.14	0.4	10
B21	2.04	0.08	0.8	10	B21	1.86	0.07	0.8	10	B21	2.01	0.08	0.8	10
B21	2.01	0.29	0.1	20	B21	1.20	0.13	0.1	20	B21	2.72	1.41	0.1	20
B21	1.88	0.09	0.4	20	B21	1.62	0.08	0.4	20	B21	1.83	0.10	0.4	20
B21	1.96	0.06	0.8	20	B21	1.88	0.06	0.8	20	B21	1.95	0.06	0.8	20
B22	4.56	0.64	0.1	5	B22	3.03	0.16	0.1	5	B22	2.51	1.87	0.1	5
B22	4.77	0.20	0.4	5	B22	3.88	0.15	0.4	5	B22	5.13	0.29	0.4	5

B22	5.06	0.17	0.8	5	B22	4.46	0.15	0.8	5	B22	5.00	0.18	0.8	5
B22	4.50	0.49	0.1	10	B22	3.30	0.16	0.1	10	B22	7.13	2.09	0.1	10
B22	5.06	0.16	0.4	10	B22	4.26	0.13	0.4	10	B22	4.98	0.19	0.4	10
B22	4.86	0.12	0.8	10	B22	4.58	0.11	0.8	10	B22	4.86	0.12	0.8	10
B22	5.07	0.40	0.1	20	B22	3.47	0.18	0.1	20	B22	8.06	1.92	0.1	20
B22	5.09	0.13	0.4	20	B22	4.55	0.12	0.4	20	B22	5.02	0.15	0.4	20
B22	5.02	0.09	0.8	20	B22	4.84	0.09	0.8	20	B22	4.98	0.09	0.8	20
Int1	1.02	0.07	0.1	5	Int1	0.99	0.07	0.1	5	Int1	0.99	0.08	0.1	5
Int1	0.97	0.08	0.4	5	Int1	0.93	0.08	0.4	5	Int1	0.91	0.08	0.4	5
Int1	0.87	0.08	0.8	5	Int1	0.81	0.08	0.8	5	Int1	0.82	0.08	0.8	5
Int1	1.02	0.05	0.1	10	Int1	1.01	0.05	0.1	10	Int1	0.96	0.06	0.1	10
Int1	1.09	0.06	0.4	10	Int1	1.06	0.06	0.4	10	Int1	1.05	0.06	0.4	10
Int1	0.98	0.06	0.8	10	Int1	0.93	0.06	0.8	10	Int1	0.92	0.06	0.8	10
Int1	1.00	0.04	0.1	20	Int1	0.99	0.04	0.1	20	Int1	1.01	0.04	0.1	20
Int1	0.96	0.04	0.4	20	Int1	0.95	0.04	0.4	20	Int1	0.95	0.04	0.4	20
Int1	1.01	0.05	0.8	20	Int1	0.99	0.05	0.8	20	Int1	0.99	0.05	0.8	20
Int2	2.07	0.08	0.1	5	Int2	2.08	0.08	0.1	5	Int2	2.08	0.08	0.1	5
Int2	1.97	0.08	0.4	5	Int2	1.95	0.08	0.4	5	Int2	1.92	0.08	0.4	5
Int2	1.90	0.09	0.8	5	Int2	1.89	0.09	0.8	5	Int2	1.91	0.09	0.8	5
Int2	2.01	0.06	0.1	10	Int2	2.00	0.06	0.1	10	Int2	1.95	0.06	0.1	10
Int2	2.10	0.06	0.4	10	Int2	2.07	0.06	0.4	10	Int2	2.06	0.06	0.4	10
Int2	1.98	0.07	0.8	10	Int2	1.94	0.07	0.8	10	Int2	1.93	0.07	0.8	10
Int2	2.03	0.05	0.1	20	Int2	2.02	0.05	0.1	20	Int2	2.06	0.05	0.1	20
Int2	1.97	0.05	0.4	20	Int2	1.96	0.05	0.4	20	Int2	1.96	0.05	0.4	20
Int2	2.00	0.05	0.8	20	Int2	1.98	0.05	0.8	20	Int2	1.98	0.06	0.8	20
Ψ1	0.00	.	0.1	5	Ψ1	0.22	0.10	0.1	5	Ψ1	0.22	0.10	0.1	5
Ψ1	0.34	0.11	0.4	5	Ψ1	0.35	0.11	0.4	5	Ψ1	0.35	0.11	0.4	5
Ψ1	0.28	0.11	0.8	5	Ψ1	0.23	0.11	0.8	5	Ψ1	0.23	0.11	0.8	5
Ψ1	0.22	0.06	0.1	10	Ψ1	0.22	0.06	0.1	10	Ψ1	0.22	0.06	0.1	10
Ψ1	0.32	0.06	0.4	10	Ψ1	0.32	0.06	0.4	10	Ψ1	0.32	0.06	0.4	10
Ψ1	0.26	0.06	0.8	10	Ψ1	0.27	0.06	0.8	10	Ψ1	0.27	0.06	0.8	10
Ψ1	0.31	0.04	0.1	20	Ψ1	0.31	0.04	0.1	20	Ψ1	0.31	0.04	0.1	20
Ψ1	0.30	0.04	0.4	20	Ψ1	0.31	0.04	0.4	20	Ψ1	0.31	0.04	0.4	20

Ψ1	0.32	0.04	0.8	20	Ψ1	0.33	0.04	0.8	20	Ψ1	0.33	0.04	0.8	20
Ψ2	1.24	0.17	0.1	5	Ψ2	1.16	0.18	0.1	5	Ψ2	1.16	0.18	0.1	5
Ψ2	0.93	0.17	0.4	5	Ψ2	1.03	0.17	0.4	5	Ψ2	1.03	0.17	0.4	5
Ψ2	1.16	0.21	0.8	5	Ψ2	1.63	0.25	0.8	5	Ψ2	1.63	0.25	0.8	5
Ψ2	0.99	0.10	0.1	10	Ψ2	0.99	0.10	0.1	10	Ψ2	0.99	0.10	0.1	10
Ψ2	1.14	0.12	0.4	10	Ψ2	1.25	0.12	0.4	10	Ψ2	1.25	0.12	0.4	10
Ψ2	0.97	0.12	0.8	10	Ψ2	1.14	0.13	0.8	10	Ψ2	1.14	0.13	0.8	10
Ψ2	0.99	0.07	0.1	20	Ψ2	1.01	0.08	0.1	20	Ψ2	1.01	0.08	0.1	20
Ψ2	1.15	0.09	0.4	20	Ψ2	1.24	0.09	0.4	20	Ψ2	1.24	0.09	0.4	20
Ψ2	1.10	0.09	0.8	20	Ψ2	1.24	0.10	0.8	20	Ψ2	1.24	0.10	0.8	20

Appendix C

SAS Code for Simulations in Chapter 3

Appendix – SAS code for large scale Simulations

This appendix shows the SAS code used for the large sample simulations of all five different generalized linear models used in Chapter 2. In order, Logit (logistic regression), Probit, Poisson (log-linear), Ordinal regression (generalized probit), and Nominal regression are shown. The code is presented in macros, with a description of macro variables for use when running the code. This is an extension of the simulations in Chapter two, and thus the code is an extension of the code seen beforehand in the Chapter 2 SAS code appendix, but here the macro is built for simulating these models n times, and at the end of the n simulations it is made to calculate statistics such as mean, mean standard error, mean lower and upper confidence limits and coverage percentages for each estimated parameter.

Logit

```
*LOGIT(RUN, IT, NJ, TAO, PSI, m);
* run= run number;
* it = iteration number, use for running different sets of parameters and potentially saving data
sets;
* nj = n per group;
* tao = x level 2 variability;
* psi = level 2 random error;
* m = number of iterations of simulation, we did 1000;


%macro LOGIT(RUN, IT, NJ, TAO, PSI, m);
%do i=1 %to &m;
data sim&IT;
/* simulate X as before */
        do i=1 to 1000;
                u&IT=rand('NORMAL', 0, &tao);
                ps&IT=rand('NORMAL', 0, sqrt(&PSI));
                do j=1 to &NJ;
                        er&IT=rand('NORMAL', 0, 1); output;
                end;
        end;
run;

data sim&IT; set sim&IT;
        d&IT=u&IT+er&IT;
/* group observed values, u's are "true means" */
                                                /* ers are level 1 scaled d's */
```

```
run;
proc means data=sim&IT noprint;
        by i; var d&IT;
        output out=MEANS;
run;
data means; set means;
        if _STAT_="MEAN";
        dbar&IT=d&IT;                                   /* group sample means */
keep i dbar&IT;
run;

data sim&IT; merge sim&IT means; by i;
        keep i j u&IT er&IT ps&IT d&IT dbar&IT;
run;

data sim&IT; set sim&IT;
        linp&IT = 1 + 1*er&IT + 2*u&IT +ps&IT;
run;
data sim&IT; set sim&IT;
        prob&IT = exp(linp&IT)/(1+exp(linp&IT)) ;
        yhat&IT = rand('BERNOULLI', prob&IT);           /* yhat for sims */
        ebar&IT= d&IT-dbar&IT;
run;
data sim&IT; set sim&IT;

ods output ParameterEstimates=trueParams&IT;
ods output CovParms=trueCovParms&IT;
proc glimmix data=sim&IT method=gauss; /* Model A */
        class i;
        model yhat&it (event=LAST) = u&IT er&IT / s dist=bernoulli link=logit  ;
        random int / subject=i ;
run;
ods output close;
proc append base=trueparamsrun&RUN data=trueparams&IT force; run;
proc append base=trueparamsrun&RUN data=truecovparms&IT force; run;

ods output ParameterEstimates=barParams&IT;
ods output CovParms=barCovParms&IT;
proc glimmix data=sim&IT method=gauss;  /* Model B */
        class i j;
        model yhat&IT(event=LAST) = dbar&IT ebar&IT / s dist=bernoulli link=logit ;
        random intercept / subject= i ;
run;
ods output close;
proc append base=barparamsrun&RUN data=barparams&IT force; run;
proc append base=barparamsrun&RUN data=barcovparms&IT force; run;
```

```
/* following is for EB analysis */
ods output CovParms=CovParms&IT;
ods output ParameterEstimates=Params&IT;
proc glimmix data=sim&IT;
        class i;
        model d&IT = / s ;
        random i;
run;

data ebset1; set covparms&IT;
        do i=1 to &nj*1000;
                where covparm="i"; do; tao=estimate; end;
                        output;
        end;
        keep tao;
run;
data ebset2; set covparms&IT;
        do i=1 to &nj*1000;
                where covparm="Residual"; do; sig=estimate; end;
                 output;
        end;
        keep sig;
run;
data ebset3; set params&IT;
        do i=1 to &nj*1000;
        int=estimate;
                 output;
        end;
        keep int;
run;
data ebset; merge ebset1 ebset2 ebset3; run;
data eb; merge ebset sim&IT; run;
data eb; set eb;
        rel= tao / (tao + (sig/ &nj )); /* reliability */
run;
data eb; set eb;
        deb&IT= (1-rel)*INT + rel*dbar&IT;  /* EB Mean */
run;
data eb; set eb;
        eeb&IT= d&IT-deb&IT;
run;

ods output ParameterEstimates=ebParams&IT;
ods output CovParms=ebCovParms&IT;
proc glimmix data=eb method=gauss;   /*  Model C */
        class i j;
```

```
	model yhat&IT(event=LAST) = deb&IT eeb&IT / s dist=bernoulli link=logit ;
	random intercept / subject= i ;
run;
ods output close;
proc append base=ebparmsrun&RUN data=ebparams&IT force; run;
proc append base=ebparmsrun&RUN data=ebcovparms&IT force; run;
%end;
/* calculate CIs, LogCIs for variances, and coverages */
data trueparamsrun&run; set trueparamsrun&run;
	ll= estimate - 1.96* stderr;
	ul= estimate + 1.96* stderr;
	lll = exp( log(estimate) - (1.96 * sqrt(1/estimate) * stderr) ) ;
	lul = exp( log(estimate) + (1.96 * sqrt(1/estimate) * stderr) ) ;
run;
data trueparamsrun&run; set trueparamsrun&run;
	if ll lt 1 and ul gt 1 then do; contain1=1; end;
	else do; contain1=0; end;
	if ll lt 2 and ul gt 2 then do; contain2=1; end;
	else do; contain2=0; end;
	if ll lt 0.2 and ul gt 0.2 then do; containp2=1; end;
	else  do; containp2=0; end;
	if ll lt 0.5 and ul gt 0.5 then do; containp5=1; end;
	else do; containp5=0; end;

	if lll lt 1 and lul gt 1 then do; lcontain1=1; end;
	else do; lcontain1=0; end;
	if lll lt 0.2 and lul gt 0.2 then do; lcontainp2=1; end;
	else  do; lcontainp2=0; end;
	if lll lt 0.5 and lul gt 0.5 then do; lcontainp5=1; end;
	else do; lcontainp5=0; end;
run;

data barparamsrun&run; set barparamsrun&run;
	ll= estimate - 1.96* stderr; /* CIs */
	ul= estimate + 1.96* stderr;
	lll = exp( log(estimate) - (1.96 * sqrt(1/estimate) * stderr) ) ;
	lul = exp( log(estimate) + (1.96 * sqrt(1/estimate) * stderr) ) ;
run;
data barparamsrun&run; set barparamsrun&run;
	if ll lt 1 and ul gt 1 then do; contain1=1; end;
	else do; contain1=0; end;
	if ll lt 2 and ul gt 2 then do; contain2=1; end;
	else do; contain2=0; end;
	if ll lt 0.2 and ul gt 0.2 then do; containp2=1; end;
	else  do; containp2=0; end;
	if ll lt 0.5 and ul gt 0.5 then do; containp5=1; end;
```

```
		else do; containp5=0; end;

		if lll lt 1 and lul gt 1 then do; lcontain1=1; end;
		else do; lcontain1=0; end;
		if lll lt 0.2 and lul gt 0.2 then do; lcontainp2=1; end;
		else  do; lcontainp2=0; end;
		if lll lt 0.5 and lul gt 0.5 then do; lcontainp5=1; end;
		else do; lcontainp5=0; end;
run;

data ebparmsrun&run; set ebparmsrun&run;
		ll= estimate - 1.96* stderr;
		ul= estimate + 1.96* stderr;

		lll = exp( log(estimate) - (1.96 * sqrt(1/estimate) * stderr) ) ;
		lul = exp( log(estimate) + (1.96 * sqrt(1/estimate) * stderr) ) ;
run;
data ebparmsrun&run; set ebparmsrun&run;
		if ll lt 1 and ul gt 1 then do; contain1=1; end;
		else do; contain1=0; end;
		if ll lt 2 and ul gt 2 then do; contain2=1; end;
		else do; contain2=0; end;
		if ll lt 0.2 and ul gt 0.2 then do; containp2=1; end;
		else  do; containp2=0; end;
		if ll lt 0.5 and ul gt 0.5 then do; containp5=1; end;
		else do; containp5=0; end;

		if lll lt 1 and lul gt 1 then do; lcontain1=1; end;
		else do; lcontain1=0; end;
		if lll lt 0.2 and lul gt 0.2 then do; lcontainp2=1; end;
		else  do; lcontainp2=0; end;
		if lll lt 0.5 and lul gt 0.5 then do; lcontainp5=1; end;
		else do; lcontainp5=0; end;
run;

proc sort data=trueparamsrun&run; by effect; run;
proc means data=trueparamsrun&run n mean std median p5 p95;
		by effect; var estimate stderr ll ul contain1 contain2 containp2 containp5
				lll lul lcontain1 lcontainp2 lcontainp5; run;
proc sort data=barparamsrun&run; by effect; run;
proc means data=barparamsrun&run n mean std median p5 p95;
		by effect; var estimate stderr ll ul contain1 contain2 containp2 containp5
				lll lul lcontain1 lcontainp2 lcontainp5; run;
proc sort data=ebparmsrun&run; by effect; run;
proc means data=ebparmsrun&run n mean std median p5 p95;
		by effect; var estimate stderr ll ul contain1 contain2 containp2 containp5
```

```
                    lll lul lcontain1 lcontainp2 lcontainp5; run;
%mend LOGIT;

/* Example */
%LOGIT(1, 1, 5, 0.1 , 0.2 , 1000);
```

Probit

```
*PROBIT(RUN, IT, NJ, TAO, PSI, m);
* run= run number;
* it = iteration number, use for running different sets of parameters and potentially saving data
sets;
* nj = n per group;
* tao = x level 2 variability;
* psi = level 2 random error;
* m = number of iterations of simulation, we did 1000;


%macro PROBIT(RUN, IT, NJ, TAO, PSI, m);
%do i=1 %to &m;
data sim&IT;
          do i=1 to 1000;
                    u&IT=rand('NORMAL', 0, &tao);
                    ps&IT=rand('NORMAL', 0, sqrt(&PSI));
                    do j=1 to &NJ;
                              er&IT=rand('NORMAL', 0, 1); output;
                    end;
          end;
run;

data sim&IT; set sim&IT;
          d&IT=u&IT+er&IT;
/* group observed values, u's are "true means" */
                                                                      /* ers are level 1 scaled d's */
run;
proc means data=sim&IT noprint;
          by i; var d&IT;
          output out=MEANS;
run;
data means; set means;
          if _STAT_="MEAN";
          dbar&IT=d&IT;                                               /* group sample means */
keep i dbar&IT;
run;
```

```
data sim&IT; merge sim&IT means; by i;
	keep i j u&IT er&IT ps&IT d&IT dbar&IT;
run;

data sim&IT; set sim&IT;
	linp&IT = 1 + 1*er&IT + 2*u&IT +ps&IT;
run;
data sim&IT; set sim&IT;
	proprob&IT = cdf('NORMAL', linp&IT, 0, 1) ;
run;
data sim&IT; set sim&IT;
	proyhat&IT = rand('BERNOULLI', proprob&IT);
	ebar&IT= d&IT-dbar&IT;
run;

ods output ParameterEstimates=trueParams&IT;
ods output CovParms=trueCovParms&IT;
proc glimmix data=sim&IT method=gauss;
	class i;
	model proyhat&it (event=LAST) = u&IT er&IT / s dist=binary link=probit  ;
	random int / subject=i ;
run;
ods output close;
proc append base=trueparamsrun&RUN data=trueparams&IT force; run;
proc append base=trueparamsrun&RUN data=truecovparms&IT force; run;

ods output ParameterEstimates=barParams&IT;
ods output CovParms=barCovParms&IT;
proc glimmix data=sim&IT method=gauss;
	class i j;
	model proyhat&IT(event=LAST) = dbar&IT ebar&IT / s dist=binary link=probit ;
	random intercept / subject= i ;
run;
ods output close;
proc append base=barparamsrun&RUN data=barparams&IT force; run;
proc append base=barparamsrun&RUN data=barcovparms&IT force; run;

ods output CovParms=CovParms&IT;
ods output ParameterEstimates=Params&IT;
proc glimmix data=sim&IT;
	class i;
	model d&IT = / s ;
	random i;
run;

data ebset1; set covparms&IT;
```

```
        do i=1 to &nj*1000;
                where covparm="i"; do; tao=estimate; end;
                        output;
        end;
        keep tao;
run;
data ebset2; set covparms&IT;
        do i=1 to &nj*1000;
                where covparm="Residual"; do; sig=estimate; end;
                 output;
        end;
        keep sig;
run;
data ebset3; set params&IT;
        do i=1 to &nj*1000;
        int=estimate;
                 output;
        end;
        keep int;
run;
data ebset; merge ebset1 ebset2 ebset3; run;
data eb; merge ebset sim&IT; run;
data eb; set eb;
        rel= tao / (tao + (sig/ &nj ));
run;
data eb; set eb;
        deb&IT= (1-rel)*INT + rel*dbar&IT;
run;
data eb; set eb;
        eeb&IT= d&IT-deb&IT;
run;

ods output ParameterEstimates=ebParams&IT;
ods output CovParms=ebCovParms&IT;
proc glimmix data=eb method=gauss;
        class i j;
        model proyhat&IT(event=LAST) = deb&IT eeb&IT / s dist=binary link=probit ;
        random intercept / subject= i ;
run;
ods output close;
proc append base=ebparmsrun&RUN data=ebparams&IT force; run;
proc append base=ebparmsrun&RUN data=ebcovparms&IT force; run;
%end;

data trueparamsrun&run; set trueparamsrun&run;
        ll= estimate - 1.96* stderr;
```

```
        ul= estimate + 1.96* stderr;
        lll = exp( log(estimate) - (1.96 * sqrt(1/estimate) * stderr) ) ;
        lul = exp( log(estimate) + (1.96 * sqrt(1/estimate) * stderr) ) ;
run;
data trueparamsrun&run; set trueparamsrun&run;
        if ll lt 1 and ul gt 1 then do; contain1=1; end;
        else do; contain1=0; end;
        if ll lt 2 and ul gt 2 then do; contain2=1; end;
        else do; contain2=0; end;
        if ll lt 0.2 and ul gt 0.2 then do; containp2=1; end;
        else  do; containp2=0; end;
        if ll lt 0.5 and ul gt 0.5 then do; containp5=1; end;
        else do; containp5=0; end;

        if lll lt 1 and lul gt 1 then do; lcontain1=1; end;
        else do; lcontain1=0; end;
        if lll lt 0.2 and lul gt 0.2 then do; lcontainp2=1; end;
        else  do; lcontainp2=0; end;
        if lll lt 0.5 and lul gt 0.5 then do; lcontainp5=1; end;
        else do; lcontainp5=0; end;
run;

data barparamsrun&run; set barparamsrun&run;
        ll= estimate - 1.96* stderr;
        ul= estimate + 1.96* stderr;
        lll = exp( log(estimate) - (1.96 * sqrt(1/estimate) * stderr) ) ;
        lul = exp( log(estimate) + (1.96 * sqrt(1/estimate) * stderr) ) ;
run;
data barparamsrun&run; set barparamsrun&run;
        if ll lt 1 and ul gt 1 then do; contain1=1; end;
        else do; contain1=0; end;
        if ll lt 2 and ul gt 2 then do; contain2=1; end;
        else do; contain2=0; end;
        if ll lt 0.2 and ul gt 0.2 then do; containp2=1; end;
        else  do; containp2=0; end;
        if ll lt 0.5 and ul gt 0.5 then do; containp5=1; end;
        else do; containp5=0; end;

        if lll lt 1 and lul gt 1 then do; lcontain1=1; end;
        else do; lcontain1=0; end;
        if lll lt 0.2 and lul gt 0.2 then do; lcontainp2=1; end;
        else  do; lcontainp2=0; end;
        if lll lt 0.5 and lul gt 0.5 then do; lcontainp5=1; end;
        else do; lcontainp5=0; end;
run;
```

```
data ebparmsrun&run; set ebparmsrun&run;
	ll= estimate - 1.96* stderr;
	ul= estimate + 1.96* stderr;

	lll = exp( log(estimate) - (1.96 * sqrt(1/estimate) * stderr) ) ;
	lul = exp( log(estimate) + (1.96 * sqrt(1/estimate) * stderr) ) ;
run;
data ebparmsrun&run; set ebparmsrun&run;
	if ll lt 1 and ul gt 1 then do; contain1=1; end;
	else do; contain1=0; end;
	if ll lt 2 and ul gt 2 then do; contain2=1; end;
	else do; contain2=0; end;
	if ll lt 0.2 and ul gt 0.2 then do; containp2=1; end;
	else  do; containp2=0; end;
	if ll lt 0.5 and ul gt 0.5 then do; containp5=1; end;
	else do; containp5=0; end;

	if lll lt 1 and lul gt 1 then do; lcontain1=1; end;
	else do; lcontain1=0; end;
	if lll lt 0.2 and lul gt 0.2 then do; lcontainp2=1; end;
	else  do; lcontainp2=0; end;
	if lll lt 0.5 and lul gt 0.5 then do; lcontainp5=1; end;
	else do; lcontainp5=0; end;
run;

proc sort data=trueparamsrun&run; by effect; run;
proc means data=trueparamsrun&run n mean std median p5 p95;
	by effect; var estimate stderr ll ul contain1 contain2 containp2 containp5
		lll lul lcontain1 lcontainp2 lcontainp5; run;
proc sort data=barparamsrun&run; by effect; run;
proc means data=barparamsrun&run n mean std median p5 p95;
	by effect; var estimate stderr ll ul contain1 contain2 containp2 containp5
		lll lul lcontain1 lcontainp2 lcontainp5; run;
proc sort data=ebparmsrun&run; by effect; run;
proc means data=ebparmsrun&run n mean std median p5 p95;
	by effect; var estimate stderr ll ul contain1 contain2 containp2 containp5
		lll lul lcontain1 lcontainp2 lcontainp5; run;
%mend PROBIT;

/* Example */
%PROBIT(1, 1, 5, 0.1 , 0.2 , 1000 );
```

Appendix D

Tabled Results for Simulations in Chapter 3

Appendix – Results of Large Scale Simulations

This appendix contains the resulting tables from the large scale simulations in Chapter 3. That is, there is a section for the Logistic regression simulations as well as one for the probit model simulations. These contain average estimates, standard errors, lower and upper confidence limits, and coverage percentages for the model parameters over all m=1000 iterations of the simulations. There are 27 sets of simulation parameters for each type of model.

1. Logistic Regression Simulations

Table D.1 Logistic Large Scale Simulations, Ψ=0.2

		Model A				Model B				Model C			
		B0	B1	B2	Ψ	B0	B1	B2	Ψ	B0	B1	B2	Ψ
	Avg Estimate	1	1	1.99	0.2	1	1.05	1	0.21	0.97	1	4.88	0.21
m=1000	Avg SE	0.04	0.04	0.37	0.07	0.04	0.08	0.05	0.07	0.09	0.04	5.76	0.07
Nj =5	Avg LL	0.92	0.92	1.26	0.15	0.92	0.88	0.91	0.16	0.8	0.91	-5.58	0.16
τ=0.1	Avg UL	1.08	1.08	2.72	0.27	1.08	1.21	1.09	0.29	1.14	1.09	17.02	0.29
Ψ=0.2	Coverage %	95.5	95.3	93.3	62.2	95.8	92.2	0	61	94.6	95.2	79.1	61.2
	Avg Estimate	1	1	2	0.2	1	1	1.45	0.29	1	1	2.02	0.29
m=1000	Avg SE	0.04	0.04	0.11	0.07	0.04	0.05	0.07	0.08	0.04	0.05	0.16	0.08
Nj =5	Avg LL	0.92	0.91	1.79	0.14	0.91	0.91	1.3	0.22	0.91	0.91	1.69	0.22
τ=0.4	Avg UL	1.08	1.08	2.21	0.27	1.08	1.09	1.59	0.38	1.08	1.09	2.34	0.38
Ψ=0.2	Coverage %	94.9	94.7	93.1	62.1	94.5	96.3	0	37.5	93.7	96.3	94	37.5
	Avg Estimate	1	1	2	0.2	1	1	1.77	0.35	1	1	2.01	0.35
m=1000	Avg SE	0.05	0.05	0.07	0.08	0.05	0.05	0.07	0.09	0.05	0.05	0.08	0.09
Nj =5	Avg LL	0.91	0.91	1.86	0.14	0.91	0.91	1.64	0.26	0.91	0.91	1.85	0.26
τ=0.8	Avg UL	1.09	1.09	2.14	0.28	1.1	1.1	1.9	0.47	1.1	1.1	2.17	0.48
Ψ=0.2	Coverage %	95.1	94.9	96	62.8	95.3	94.8	6.5	18.6	94.9	94.8	96.8	18.6
	Avg Estimate	1	1	2.01	0.2	1	1	1.09	0.21	1	1	2.4	0.21
m=1000	Avg SE	0.03	0.03	0.28	0.04	0.03	0.03	0.09	0.04	0.03	0.03	1.7	0.04
Nj =10	Avg LL	0.94	0.94	1.46	0.17	0.94	0.94	0.92	0.18	0.93	0.94	-0.84	0.18
τ=0.1	Avg UL	1.06	1.06	2.56	0.23	1.06	1.06	1.26	0.25	1.07	1.06	5.87	0.25
Ψ=0.2	Coverage %	94.4	95.2	95.5	60.2	94.2	95.8	0	60.7	93.5	95.8	90.9	60.7

	Avg Estimate	1	1	2	0.2	1	1	1.62	0.26	1	1	2	0.26
m=1000	Avg SE	0.03	0.03	0.08	0.04	0.03	0.03	0.06	0.04	0.03	0.03	0.1	0.04
Nj =10	Avg LL	0.94	0.94	1.84	0.17	0.94	0.94	1.49	0.22	0.94	0.94	1.8	0.22
τ=0.4	Avg UL	1.06	1.06	2.15	0.24	1.06	1.06	1.74	0.31	1.06	1.06	2.2	0.31
Ψ=0.2	Coverage %	96.3	95	96	62.4	95.9	95.4	0	27.1	95.9	95.4	93.8	27.1
	Avg Estimate	1	1	2	0.2	1	1	1.86	0.28	1	1	2	0.28
m=1000	Avg SE	0.03	0.03	0.05	0.04	0.03	0.03	0.05	0.05	0.03	0.03	0.06	0.05
Nj =10	Avg LL	0.93	0.94	1.9	0.16	0.93	0.94	1.77	0.24	0.93	0.94	1.89	0.24
τ=0.8	Avg UL	1.06	1.06	2.1	0.24	1.07	1.06	1.96	0.34	1.07	1.06	2.11	0.34
Ψ=0.2	Coverage %	95.5	93.9	95.5	62.3	95.1	93.6	23.5	17.2	94.8	93.6	94.9	37.8
	Avg Estimate	1	1	1.99	0.2	1	1	1.17	0.21	1	1	2.05	0.21
m=1000	Avg SE	0.02	0.02	0.22	0.02	0.02	0.02	0.09	0.02	0.02	0.02	0.6	0.02
Nj =20	Avg LL	0.96	0.96	1.56	0.18	0.96	0.96	0.98	0.19	0.95	0.96	0.88	0.19
τ=0.1	Avg UL	1.05	1.04	2.43	0.22	1.05	1.04	1.35	0.23	1.05	1.04	3.22	0.23
Ψ=0.2	Coverage %	94.9	95	95.3	64.6	94.7	94.9	0	61.7	94.5	94.9	94.2	61.7
	Avg Estimate	1	1	2	0.2	1	1	1.76	0.23	1	1	2	0.23
m=1000	Avg SE	0.02	0.02	0.06	0.02	0.02	0.02	0.06	0.03	0.02	0.02	0.07	0.03
Nj =20	Avg LL	0.95	0.96	1.88	0.18	0.95	0.96	1.65	0.21	0.95	0.96	1.86	0.21
τ=0.4	Avg UL	1.05	1.04	2.12	0.22	1.05	1.04	1.87	0.26	1.05	1.04	2.14	0.26
Ψ=0.2	Coverage %	93.6	94.3	93.8	65	93.6	95.1	1.1	29.5	93.3	95.1	94.6	29.5
	Avg Estimate	1	1	2	0.2	1	1	1.93	0.24	1	1	2	0.24
m=1000	Avg SE	0.03	0.02	0.04	0.03	0.03	0.02	0.04	0.03	0.03	0.02	0.04	0.03
Nj =20	Avg LL	0.95	0.96	1.92	0.18	0.95	0.96	1.85	0.22	0.95	0.96	1.92	0.22
τ=0.8	Avg UL	1.05	1.05	2.08	0.22	1.05	1.05	2	0.27	1.05	1.05	2.08	0.27
Ψ=0.2	Coverage %	94.8	93.8	94.3	62	94.7	94	53.2	22.5	94.7	94	94.6	22.5

Table D.2 Logistic Large Scale Simulations, Ψ=0.5

		Model A				Model B				Model C			
		B0	B1	B2	PSI	B0	B1	B2	PSI	B0	B1	B2	PSI
	Avg Estimate	1	1	2	0.5	1	1	1.05	0.51	1.02	1	4.32	0.51
m=1000	Avg SE	0.04	0.04	0.42	0.09	0.04	0.05	0.09	0.09	0.09	0.05	6.08	0.09
Nj =5	Avg LL	0.91	0.92	1.18	0.39	0.91	0.91	0.86	0.4	0.83	0.91	-6.71	0.4
τ =0.1	Avg UL	1.09	1.06	2.81	0.65	1.09	1.09	1.23	0.66	1.2	1.09	17.13	0.66
Ψ =0.5	Coverage %	94	95.6	93.8	84.4	94.5	95.1	0	84.9	94.2	95.2	78.8	84.9
	Avg Estimate	1	1	2	0.5	1	1	1.44	0.59	1	1	2	0.59
m=1000	Avg SE	0.05	0.04	0.12	0.09	0.05	0.05	0.08	0.1	0.05	0.05	0.18	0.1
Nj =5	Avg LL	0.91	0.92	1.77	0.38	0.91	0.91	1.29	0.45	0.91	0.91	1.65	0.45
τ =0.4	Avg UL	1.09	1.09	2.23	0.65	1.09	1.1	1.6	0.76	1.09	1.09	2.35	0.76
Ψ =0.5	Coverage %	96.2	94.8	95.3	80.3	96.2	95.3	0	68.9	95	95.3	92.9	68.8
	Avg Estimate	1	1	2	0.5	1	1	1.76	0.65	1	1	2	0.65
m=1000	Avg SE	0.05	0.05	0.08	0.1	0.05	0.05	0.07	0.11	0.05	0.05	0.09	0.11
Nj =5	Avg LL	0.9	0.91	1.85	0.37	0.89	0.9	1.62	0.49	0.9	0.9	1.83	0.49
τ =0.8	Avg UL	1.1	1.09	2.15	0.66	1.1	1.1	1.9	0.86	1.1	1.1	2.17	0.86
Ψ =0.5	Coverage %	94.5	93.2	95.5	81.3	94.6	93	8.6	54.1	94.6	93	95.6	54.1
	Avg Estimate	1	1	2	0.5	1	1	1.1	0.51	1.53	1	43.07	0.51
m=1000	Avg SE	0.03	0.03	0.33	0.06	0.04	0.03	0.1	0.06	0.04	0.03	2.14	0.06
Nj =10	Avg LL	0.93	0.94	1.35	0.43	0.93	0.94	0.9	0.44	1.44	0.94	4.012	0.44
τ =0.1	Avg UL	1.07	1.06	2.66	0.58	1.07	1.06	1.3	0.59	1.62	1.06	48.5	0.59
Ψ =0.5	Coverage %	94.8	95.8	96.2	82.6	95.1	96	0	83.4	93.7	96	92.4	83.4
	Avg Estimate	1	1	2	0.5	1	1	1.62	0.56	1	1	2.01	0.56
m=1000	Avg SE	0.04	0.03	0.09	0.06	0.04	0.03	0.07	0.06	0.04	0.03	0.12	0.06
Nj =10	Avg LL	0.93	0.94	1.82	0.42	0.93	0.94	1.47	0.48	0.93	0.94	1.78	0.48
τ =0.4	Avg UL	1.07	1.06	2.18	0.58	1.07	1.06	1.76	0.65	1.07	1.06	2.24	0.65
Ψ =0.5	Coverage %	96.1	95	93.9	84.2	96.6	95.5	0.1	67.3	95.6	95.5	93.5	67.3
	Avg Estimate	1	1	2	0.5	1	1	1.87	0.59	1	1	2	0.59
m=1000	Avg SE	0.04	0.03	0.06	0.06	0.04	0.03	0.06	0.07	0.04	0.03	0.06	0.07
Nj =10	Avg LL	0.93	0.94	1.89	0.42	0.92	0.94	1.76	0.5	0.92	0.94	1.88	0.5
τ =0.8	Avg UL	1.08	1.06	2.12	0.6	1.08	1.07	1.98	0.7	1.08	1.007	2.13	0.7

Ψ =0.5	Coverage %	93.8	94.9	94.4	82.7	93.5	95	33.9	52.6	93.1	95	94	52.6
	Avg Estimate	1	1	2	0.5	1	1	1.17	0.51	1	1	2.1	0.51
m=1000	Avg SE	0.03	0.02	0.288	0.04	0.03	0.02	0.12	0.04	0.03	0.02	0.78	0.04
Nj =20	Avg LL	0.94	0.96	1.44	0.45	0.94	0.96	0.94	0.45	0.94	0.96	0.59	0.45
τ =0.1	Avg UL	1.05	1.04	2.56	0.55	1.06	1.04	1.4	0.56	1.06	1.04	3.6	0.56
Ψ =0.5	Coverage %	95.5	94.2	95.5	83.7	95.3	94.3	0	84.2	94.7	94.3	94.5	84.1
	Avg Estimate	1	1	2	0.5	1	1	1.77	0.54	1	1	2.01	0.54
m=1000	Avg SE	0.03	0.02	0.088	0.04	0.03	0.02	0.07	0.04	0.03	0.02	0.09	0.04
Nj =20	Avg LL	0.94	0.96	1.85	0.45	0.94	0.96	1.63	0.48	0.94	0.96	1.83	0.48
τ =0.4	Avg UL	1.06	1.04	2.15	0.56	1.06	1.04	1.9	0.6	1.06	1.04	2.18	0.6
Ψ =0.5	Coverage %	94.2	94.4	93.8	83.3	94.2	94.2	7.9	68	94	94.2	94	68
	Avg Estimate	1	1	2	0.5	1	1	1.93	0.54	1	1	2	0.54
m=1000	Avg SE	0.03	0.02	0.05	0.04	0.03	0.02	0.05	0.04	0.03	0.02	0.05	0.04
Nj =20	Avg LL	0.94	0.96	1.91	0.44	0.93	0.95	1.84	0.48	0.93	0.5	1.91	0.48
τ =0.8	Avg UL	1.06	1.04	2.09	0.56	1.06	1.04	2.02	0.61	1.06	1.04	2.1	0.61
Ψ =0.5	Coverage %	94.9	94.1	95.3	83.5	95	93.3	63.5	67.7	94.7	93.3	95.3	67.7

Table D.3 Logistic Large Scale Simulations, Ψ=1.0

		Model A				Model B				Model C			
		B0	B1	B2	Ψ	B0	B1	B2	Ψ	B0	B1	B2	Ψ
	Avg Estimate	1	1	2	1	1	1	1.05	1.01	1.01	1	2.32	1.01
m=1000	Avg SE	0.05	0.04	0.48	0.13	0.05	0.05	0.11	0.13	0.08	0.05	4.93	0.13
Nj =5	Avg LL	0.9	0.92	1.06	0.78	0.9	0.91	0.84	0.79	0.85	0.91	-6.86	0.79
τ=0.1	Avg UL	1.1	1.09	2.95	1.29	1.1	1.1	1.26	1.3	1.16	1.1	14.46	1.3
Ψ=1.0	Coverage %	95.9	94.6	94.8	95.1	95.6	93.9	0	95.3	95.7	94.2	78.3	95.3
	Avg Estimate	1	1	2.01	1	1	1	1.45	1.09	1	1	2.02	1.09
m=1000	Avg SE	0.05	0.05	0.13	0.13	0.05	0.05	0.09	0.14	0.05	0.05	0.2	0.14
Nj =5	Avg LL	0.9	0.91	1.75	0.77	0.9	0.91	1.3	0.84	0.9	0.91	1.62	0.84
τ=0.4	Avg UL	1.1	1.09	2.27	1.29	1.1	1.1	1.63	1.41	1.11	1.1	2.41	1.41
Ψ=1.0	Coverage %	94.1	95.3	94.1	95.9	94.3	94.4	0.1	94	93.9	94.4	95.2	94
	Avg Estimate	1	1	2	1	1	1	1.76	1.16	1	1	2	1.16

m=1000	Avg SE	0.06	0.05	0.08	0.14	0.06	0.05	0.08	0.15	0.06	0.05	0.1	0.15
Nj =5	Avg LL	0.89	0.9	1.84	0.76	0.89	0.9	1.62	0.87	0.89	0.9	1.82	0.87
τ=0.8	Avg UL	1.11	1.09	2.17	1.32	1.12	1.1	1.91	1.53	1.12	1.1	2.19	1.53
Ψ=1.0	Coverage %	95.4	95.7	95.1	94.7	95.9	94.9	14.4	85.6	96	94.9	95.2	85.6
	Avg Estimate	1	1	1.99	1	1	1	1.1	1.01	1	1	2.54	1.01
m=1000	Avg SE	0.04	0.03	0.41	0.09	0.04	0.03	0.12	0.09	0.05	0.03	2.3	0.09
Nj =10	Avg LL	0.92	0.94	1.19	0.83	0.92	0.94	0.85	0.85	0.91	0.94	-1.93	0.85
τ=0.1	Avg UL	1.08	1.06	2.79	1.16	1.08	1.06	1.34	1.19	1.1	1.06	7.08	1.19
Ψ=1.0	Coverage %	94.7	96.1	95.3	96.1	94.9	95.6	0	96.4	93.7	95.7	93.3	96.4
	Avg Estimate	1	1	2	1	1	1	1.61	1.06	1	1	2	1.06
m=1000	Avg SE	0.04	0.03	0.11	0.09	0.04	0.03	0.09	0.09	0.04	0.03	0.14	0.09
Nj =10	Avg LL	0.92	0.94	1.79	0.84	0.92	0.94	1.44	0.89	0.92	0.94	1.73	0.89
τ=0.4	Avg UL	1.09	1.06	2.21	1.19	1.09	1.06	1.79	1.26	1.09	1.06	2.28	1.26
Ψ=1.0	Coverage %	95.2	94.5	93.1	94.9	94.8	94.5	1.3	90.8	94.7	94.5	94.3	90.8
	Avg Estimate	1	1	2	1	1	1	1.86	1.08	1	1	2	1.08
m=1000	Avg SE	0.05	0.03	0.07	0.09	0.05	0.03	0.06	0.1	0.05	0.03	0.07	0.1
Nj =10	Avg LL	0.91	0.93	1.87	0.83	0.91	0.93	1.74	0.9	0.91	0.93	1.86	0.9
τ=0.8	Avg UL	1.09	1.06	2.13	1.2	1.09	1.06	1.99	1.3	1.09	1.06	2.14	1.3
Ψ=1.0	Coverage %	95.1	96	95.4	95	94.3	95.6	42.1	92.2	94.5	95.6	94.8	92.2
	Avg Estimate	1	1	1.99	1	1	1	1.17	1.01	1	1	2.12	1.01
m=1000	Avg SE	0.04	0.02	0.37	0.07	0.04	0.02	0.15	0.07	0.04	0.02	0.99	0.06
Nj =20	Avg LL	0.93	0.96	1.27	0.88	0.93	0.96	0.88	0.89	0.92	0.96	0.18	0.88
τ=0.1	Avg UL	1.07	1.04	2.71	1.14	1.07	1.04	1.46	1.14	1.07	1.04	4.06	1.14
Ψ=1.0	Coverage %	95.3	93.9	95.6	95.8	95.6	93.5	0	95.9	94.5	93.5	95	95.9
	Avg Estimate	1	1	2	1	1	1	1.76	1.04	1	1	2	1.04
m=1000	Avg SE	0.04	0.02	0.1	0.07	0.04	0.02	0.08	0.07	0.04	0.02	0.11	0.07
Nj =20	Avg LL	0.92	0.96	1.81	0.88	0.92	0.96	1.6	0.91	0.92	0.96	1.79	0.91
τ=0.4	Avg UL	1.07	1.04	2.19	1.14	1.07	1.04	1.93	1.18	1.07	1.04	2.22	1.18
Ψ=1.0	Coverage %	95.3	94.7	94.8	94.2	95.2	94.9	21.7	93.1	95.3	94.9	94.4	93.1
	Avg Estimate	1	1	2	1	1	1	1.93	1.05	1	1	2	1.05
m=1000	Avg SE	0.04	0.02	0.06	0.07	0.04	0.02	0.05	0.07	0.04	0.02	0.06	0.07
Nj =20	Avg LL	0.92	0.96	1.89	0.87	0.92	0.95	1.82	0.91	0.92	0.95	1.89	0.91
τ=0.8	Avg UL	1.08	1.05	2.11	1.15	1.08	1.05	2.04	1.21	1.08	1.05	2.12	1.21

Ψ=1.0	Coverage %	95.2	95	95.3	94.1	95.6	94.8	71.2	91.3	96	94.8	95.1	91.3

2. Probit Model Simulations

Table D.4 Probit Large Scale Simulations, Ψ=0.2

		Model A				Model B				Model C			
		B0	B1	B2	Ψ	B0	B1	B2	Ψ	B0	B1	B2	Ψ
	Avg Estimate	1	1	2	0.2	1	1	1.05	0.21	1	1	2.22	0.21
m=1000	Avg SE	0.03	0.03	0.28	0.04	0.03	0.04	0.07	0.04	0.05	0.04	2.998	0.04
Nj =5	Avg LL	0.93	0.93	1.45	0.12	0.93	0.93	0.82	0.17	0.9	0.93	-3.11	0.17
τ=0.1	Avg UL	1.07	1.07	2.54	0.28	1.07	1.07	1.18	0.25	1.1	1.07	8.59	0.25
Ψ=0.2	Coverage %	95.4	95.5	93.9	95.2	95.8	95.2	0	61.3	93	95	74.4	61.2
	Avg Estimate	1	1	2	0.2	1	1	1.45	0.29	1	1	2.01	0.29
m=1000	Avg SE	0.04	0.04	0.09	0.04	0.04	0.04	0.06	0.05	0.04	0.04	0.13	0.05
Nj =5	Avg LL	0.93	0.93	1.83	0.17	0.93	0.93	1.35	0.24	0.93	0.93	1.75	0.24
τ=0.4	Avg UL	1.07	1.07	2.18	0.24	1.07	1.07	1.57	0.34	1.07	1.07	2.27	0.34
Ψ=0.2	Coverage %	95.1	95.3	95	62.9	95.3	94.6	0	15.1	94.5	94.6	91.8	15.1
	Avg Estimate	1	1	1.99	0.2	1	1	1.76	0.35	1	1	2	0.35
m=1000	Avg SE	0.04	0.04	0.07	0.05	0.04	0.04	0.06	0.06	0.04	0.04	0.08	0.06
Nj =5	Avg LL	0.92	0.92	1.86	0.17	0.92	0.92	1.63	0.29	0.91	0.92	1.85	0.29
τ=0.8	Avg UL	1.08	1.08	2.13	0.25	1.08	1.08	1.89	0.43	1.08	1.08	2.15	0.43
Ψ=0.2	Coverage %	95.2	93.1	94.6	64.5	95.8	95.4	5	2.4	95.8	95.4	94.2	2.4
	Avg Estimate	1	1	2.01	0.2	1	1	1.1	0.21	1	1	3.14	0.21
m=1000	Avg SE	0.02	0.02	0.22	0.02	0.03	0.02	0.07	0.02	0.03	0.02	1.51	0.02
Nj =10	Avg LL	0.95	0.96	1.58	0.18	0.95	0.96	0.96	0.19	0.93	0.96	0.26	0.19
τ=0.1	Avg UL	1.05	1.05	2.45	0.22	1.05	1.05	1.23	0.23	1.06	1.05	6.17	0.23
Ψ=0.2	Coverage %	96.8	95.1	95.1	64.3	96.5	95.1	0	61.8	93.5	95.2	88.9	61.8
	Avg Estimate	1	1	2	0.2	1	1	1.62	0.26	1	1	2.01	0.26
m=1000	Avg SE	0.03	0.02	0.07	0.02	0.03	0.02	0.06	0.03	0.03	0.02	0.09	0.03
Nj =10	Avg LL	0.95	0.95	1.87	0.18	0.95	0.95	1.51	0.23	0.95	0.95	1.84	0.23
τ=0.4	Avg UL	1.05	1.05	2.13	0.22	1.05	1.05	1.73	0.29	1.05	1.05	2.17	0.29
Ψ=0.2	Coverage %	94.6	94.6	94.3	60	95.2	94.8	0	9.4	93.9	94.8	94	9.4

	Avg Estimate	0.99	0.99	1.99	0.2	1	1	1.86	0.29	1	1	1.99	0.29
m=1000	Avg SE	0.03	0.03	0.05	0.03	0.03	0.03	0.05	0.03	0.03	0.03	0.05	0.03
Nj =10	Avg LL	0.94	0.94	1.89	0.18	0.93	0.94	1.76	0.25	0.93	0.94	1.89	0.25
τ=0.8	Avg UL	1.05	1.05	2.09	0.23	1.06	1.05	1.95	0.32	1.06	1.05	2.1	0.32
Ψ=0.2	Coverage %	93.5	93.9	92.6	61.2	93.4	94.9	16.4	3.3	93	94.9	93.5	3.3
	Avg Estimate	1	1	2	0.2	1	1	1.17	0.21	1	1	2.1	0.21
m=1000	Avg SE	0.02	0.02	0.19	0.02	0.02	0.02	0.08	0.02	0.02	0.02	0.52	0.02
Nj =20	Avg LL	0.96	0.97	1.64	0.19	0.96	0.97	1.02	0.19	0.96	0.97	1.09	0.19
τ=0.1	Avg UL	1.04	1.03	2.37	0.21	1.04	1.03	1.32	0.22	1.04	1.03	3.11	0.22
Ψ=0.2	Coverage %	95.3	94.1	96.4	62.7	95.6	94.5	0	56.3	93.7	94.5	91.1	56.3
	Avg Estimate	1	1	2	0.2	1	1	1.76	0.24	1	1	2	0.24
m=1000	Avg SE	0.02	0.02	0.05	0.02	0.02	0.02	0.05	0.02	0.02	0.02	0.06	0.02
Nj =20	Avg LL	0.96	0.97	1.9	0.19	0.96	0.97	1.67	0.22	0.96	0.97	1.88	0.22
τ=0.4	Avg UL	1.04	1.03	2.11	0.22	1.04	1.03	1.86	0.26	1.04	1.03	2.13	0.26
Ψ=0.2	Coverage %	95.6	95.3	96.4	61.1	94.8	95.6	0.3	10.1	94.9	95.6	95	10.1
	Avg Estimate	0.99	1	1.99	0.2	1	1	1.92	0.25	1	1	2	0.25
m=1000	Avg SE	0.02	0.02	0.04	0.02	0.02	0.02	0.04	0.02	0.02	0.02	0.04	0.02
Nj =20	Avg LL	0.95	0.96	1.92	0.18	0.95	0.96	1.85	0.23	0.95	0.96	1.92	0.23
τ=0.8	Avg UL	1.04	1.03	2.07	0.22	1.04	1.03	2	0.27	1.05	1.03	2.08	0.27
Ψ=0.2	Coverage %	94.2	97.5	91.9	63.3	94.5	95.1	46.7	9	94.2	95.1	95.5	9

Table D.5 Probit Large Scale Simulations, Ψ=0.5

		Model A				Model B				Model C			
		B0	B1	B2	Ψ	B0	B1	B2	Ψ	B0	B1	B2	Ψ
	Avg Estimate	1	1	1.99	0.5	1	1	1.05	0.51	1	1	2.99	0.51
m=1000	Avg SE	0.04	0.04	0.34	0.06	0.04	0.04	0.08	0.06	0.07	0.04	5.41	0.06
Nj =5	Avg LL	0.92	0.93	1.33	0.42	0.92	0.93	0.89	0.43	0.87	0.93	-7.06	0.43
τ=0.1	Avg UL	1.08	1.07	2.64	0.6	1.08	1.07	1.2	0.61	1.15	1.07	14.15	0.61
Ψ=0.5	Coverage %	94.8	96.5	95.4	84.8	94.8	95.4	0	85.1	93.6	95.4	77	85
	Avg Estimate	1	1	2.01	0.5	1	1	1.45	0.59	1	1	2.02	0.59

m=1000	Avg SE	0.04	0.04	0.1	0.07	0.04	0.04	0.07	0.07	0.04	0.04	0.15	0.07
Nj =5	Avg LL	0.92	0.93	1.81	0.42	0.92	0.93	1.31	0.49	0.92	0.93	1.12	0.49
τ=0.4	Avg UL	1.08	1.08	2.21	0.6	1.09	1.08	1.59	0.71	1.09	1.08	2.32	0.71
Ψ=0.5	Coverage %	94.7	95.7	95.2	84.1	94.7	94.8	0	58.7	93.6	94.8	92.7	58.7
	Avg Estimate	1	1	2	0.5	1	1	1.76	0.65	1	1	2	0.65
m=1000	Avg SE	0.05	0.04	0.08	0.07	0.05	0.04	0.07	0.08	0.05	0.04	0.09	0.08
Nj =5	Avg LL	0.91	0.92	1.85	0.41	0.91	0.92	1.62	0.53	0.91	0.92	1.83	0.53
τ=0.8	Avg UL	1.09	1.08	2.15	0.61	1.09	1.08	1.9	0.81	1.09	1.08	2.17	0.81
Ψ=0.5	Coverage %	94.2	95.7	94.8	83.5	93.3	95.6	12.1	31.8	93.9	95.6	94	31.8
	Avg Estimate	1	1	2	0.5	1	1	1.09	0.51	1	1	2.33	0.51
m=1000	Avg SE	0.03	0.02	0.29	0.04	0.03	0.02	0.09	0.04	0.04	0.02	1.65	0.04
Nj =10	Avg LL	0.94	0.95	1.44	0.44	0.94	0.95	0.92	0.45	0.93	0.95	-0.83	0.45
τ=0.1	Avg UL	1.06	1.05	2.56	0.56	1.06	1.05	1.26	0.57	1.07	1.05	5.62	0.57
Ψ=0.5	Coverage %	94.5	94.1	95.8	85	94.6	94.1	0	84.5	92.2	94	89.1	84.4
	Avg Estimate	1	1	2	0.5	1	1	1.62	0.56	1	1	2.01	0.56
m=1000	Avg SE	0.03	0.02	0.08	0.04	0.03	0.03	0.07	0.05	0.03	0.03	0.11	0.05
Nj =10	Avg LL	0.94	0.95	1.84	0.44	0.94	0.95	1.49	0.49	0.94	0.95	1.8	0.49
τ=0.4	Avg UL	1.07	1.05	2.17	0.56	1.07	1.05	1.75	0.63	1.07	1.05	2.22	0.63
Ψ=0.5	Coverage %	94	95.8	94.5	83.2	94.1	95.3	0.1	57.5	94.7	95.3	93	57.5
	Avg Estimate	1	1	2	0.5	1	1	1.87	0.58	1	1	2	0.58
m=1000	Avg SE	0.04	0.03	0.06	0.05	0.04	0.03	0.06	0.05	0.04	0.03	0.06	0.05
Nj =10	Avg LL	0.93	0.95	1.89	0.44	0.93	0.95	1.76	0.51	0.93	0.95	1.88	0.51
τ=0.8	Avg UL	1.07	1.05	2.11	0.57	1.07	1.05	1.98	0.67	1.07	1.05	2.12	0.67
Ψ=0.5	Coverage %	96.2	95.3	95.9	83.9	95.2	95.9	33.1	43	94.3	95.9	95.5	43
	Avg Estimate	1	1	2	0.5	1	1	1.17	0.51	1	1	2.08	0.51
m=1000	Avg SE	0.03	0.02	0.26	0.03	0.03	0.02	0.11	0.03	0.03	0.02	0.69	0.03
Nj =20	Avg LL	0.95	0.97	1.5	0.45	0.95	0.97	0.96	0.46	0.95	0.97	0.73	0.46
τ=0.1	Avg UL	1.05	1.03	2.5	0.54	1.05	1.03	1.38	0.55	1.06	1.03	3.43	0.55
Ψ=0.5	Coverage %	95.5	95.5	95.5	83	95.8	95.7	0	82	94.7	95.7	93.3	82
	Avg Estimate	1	1	2	0.5	1	1	1.77	0.53	1	1	2.01	0.53
m=1000	Avg SE	0.03	0.02	0.07	0.03	0.03	0.02	0.06	0.04	0.03	0.02	0.08	0.04
Nj =20	Avg LL	0.95	0.97	1.87	0.45	0.95	0.97	1.64	0.49	0.95	0.97	1.85	0.49
τ=0.4	Avg UL	1.05	1.03	2.14	0.54	1.06	1.03	1.89	0.59	1.06	1.03	2.17	0.59

Ψ=0.5	Coverage %	94.8	94.8	94.6	84.8	94.6	94.9	4.5	68.7	94.4	94.9	95.1	68.7
	Avg Estimate	1	1	2	0.5	1	1	1.93	0.55	1	1	2	0.55
m=1000	Avg SE	0.03	0.02	0.05	0.04	0.03	0.02	0.05	0.04	0.03	0.02	0.05	0.04
Nj =20	Avg LL	0.94	0.96	1.91	0.45	0.94	0.96	1.84	0.49	0.94	0.96	1.9	0.49
τ=0.8	Avg UL	1.06	1.04	2.09	0.55	1.06	1.04	2.02	0.61	1.06	1.04	2.09	0.61
Ψ=0.5	Coverage %	94.7	93.3	93.6	81.3	94.8	92.9	62	59.9	94.7	92.9	94.5	59.9

Table D.6 Probit Large Scale Simulations, Ψ=1.0

		Model A				Model B				Model C			
		B0	B1	B2	Ψ	B0	B1	B2	Ψ	B0	B1	B2	Ψ
	Avg Estimate	1	1	2	1	1	1	1.05	1	1	1	3.62	1
m=1000	Avg SE	0.05	0.04	0.41	0.1	0.05	0.04	0.09	0.1	0.16	0.04	15.59	0.1
Nj =5	Avg LL	0.91	0.93	1.19	0.81	0.91	0.93	0.88	0.82	0.68	0.93	-24.3	0.82
τ=0.1	Avg UL	1.09	1.08	2.81	1.22	1.09	1.08	1.23	1.23	1.31	1.08	32.89	1.23
Ψ=1.0	Coverage %	94.6	95.4	94.4	94.5	94.6	94.9	0	95.1	94.2	95.1	77.5	95
	Avg Estimate	1	1	2	1	1	1	1.44	1.09	1	1	2	1.09
m=1000	Avg SE	0.05	0.04	0.12	0.11	0.05	0.04	0.08	0.11	0.05	0.04	0.18	0.11
Nj =5	Avg LL	0.9	0.92	1.76	0.81	0.9	0.92	1.28	0.88	0.9	0.92	1.65	0.88
τ=0.4	Avg UL	1.09	1.08	2.24	1.23	1.1	1.08	1.61	1.34	1.1	1.08	2.35	1.34
Ψ=1.0	Coverage %	94	95.5	95	94.6	94.2	94.9	0	91.7	94.3	94.9	92.7	91.7
	Avg Estimate	1	1	2	1	1	1	1.76	1.16	1	1	2	1.16
m=1000	Avg SE	0.05	0.04	0.09	0.12	0.06	0.04	0.08	0.13	0.06	0.04	0.1	0.13
Nj =5	Avg LL	0.89	0.92	1.83	0.8	0.89	0.91	1.61	0.91	0.89	0.91	1.81	0.91
τ=0.8	Avg UL	1.1	1.08	2.17	1.26	1.11	1.09	1.92	1.47	1.11	1.09	2.19	1.47
Ψ=1.0	Coverage %	95.5	94.6	95.1	95.4	94.9	94.2	19.2	82.5	95.4	94.2	93.6	82.5
	Avg Estimate	1	1	2	1	1	1	1.09	1.01	1	1	2.63	1
m=1000	Avg SE	0.04	0.02	0.4	0.07	0.04	0.03	0.11	0.07	0.05	0.03	2.48	0.07
Nj =10	Avg LL	0.92	0.95	1.28	0.86	0.92	0.95	0.87	0.87	0.9	0.95	-2.15	0.87
τ=0.1	Avg UL	1.08	1.05	2.73	1.15	1.08	1.05	1.31	1.16	1.1	1.05	7.57	0.16
Ψ=1.0	Coverage %	94.9	96.1	94.5	94.1	94.9	95.9	0	95.4	93.6	95.8	90.3	95.4
	Avg Estimate	1	1	2	0.99	1	1	1.61	1.06	1	1	2	1.06

m=1000	Avg SE	0.04	0.03	0.1	0.08	0.04	0.03	0.08	0.08	0.04	0.03	0.13	0.08
Nj =10	Avg LL	0.92	0.95	1.8	0.86	0.92	0.95	1.45	0.91	0.92	0.95	1.74	0.91
τ=0.4	Avg UL	1.08	1.05	2.2	1.15	1.08	1.05	1.77	1.23	1.08	1.05	2.26	1.23
Ψ=1.0	Coverage %	95.7	94.3	95	94.8	95.5	94.6	0.3	93.2	95.2	94.6	94.7	93.2
	Avg Estimate	1	1	2	0.99	1	1	1.86	1.08	1	1	2	1.08
m=1000	Avg SE	0.04	0.03	0.07	0.08	0.04	0.03	0.06	0.09	0.04	0.03	0.07	0.09
Nj =10	Avg LL	0.92	0.94	1.86	0.84	0.91	0.94	1.74	0.91	0.91	0.94	1.85	0.91
τ=0.8	Avg UL	1.08	1.05	2.13	1.17	1.09	1.05	1.99	1.27	1.09	1.05	2.14	1.27
Ψ=1.0	Coverage %	94.3	95.2	96.1	94.2	94	95.3	42.9	89.8	94.2	95.3	95.5	89.8
	Avg Estimate	1	1	1.98	1	1	1	1.16	1	1	1	2.06	1
m=1000	Avg SE	0.04	0.02	0.34	0.06	0.04	0.02	0.14	0.06	0.04	0.02	0.92	0.06
Nj =20	Avg LL	0.93	0.97	1.3	0.89	0.93	0.97	0.89	0.89	0.93	0.97	0.26	0.89
τ=0.1	Avg UL	1.07	1.03	2.65	1.12	1.07	1.03	1.44	1.13	1.07	1.03	3.86	1.13
Ψ=1.0	Coverage %	93.6	96.2	95.3	94.2	94.3	95.8	0.1	94.6	93.8	95.8	94.6	94.6
	Avg Estimate	1	1	2	0.99	1	1	1.76	1.03	1	1	2	1.03
m=1000	Avg SE	0.04	0.02	0.09	0.06	0.04	0.02	0.08	0.06	0.04	0.02	0.11	0.06
Nj =20	Avg LL	0.93	0.97	1.82	0.88	0.93	0.97	1.6	0.91	0.93	0.97	1.79	0.91
τ=0.4	Avg UL	1.07	1.03	2.18	1.12	1.07	1.03	1.92	1.16	1.07	1.03	2.21	1.16
Ψ=1.0	Coverage %	95	94.1	95.2	94.3	95.3	94.1	18.2	94.2	95.2	94.1	95	94.2
	Avg Estimate	1	1	2	1	1	1	1.93	1.04	1	1	2	1.04
m=1000	Avg SE	0.04	0.02	0.06	0.07	0.04	0.02	0.06	0.07	0.04	0.02	0.06	0.07
Nj=20	Avg LL	0.92	0.96	1.89	0.87	0.92	0.96	1.82	0.91	0.92	0.96	1.88	0.91
τ=0.8	Avg UL	1.08	1.04	2.11	1.13	1.08	1.04	2.04	1.19	1.08	1.04	2.12	1.19
Ψ=1.0	Coverage %	95.8	94.5	95.3	94.5	96.1	94.7	74.2	92.9	96	94.7	95.2	92.9